STATISTICAL ARBITRAGE WITH SYNTHETIC EQUITY INDEX SWAPS FOR 130/30 PRODUCTS

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Abstract

Statistical Arbitrage (SA) with synthetic index swaps for 130/30 products is one of the most innovative areas of quantitative asset management. In our analysis, we will provide a definition of SA placing it in its theoretical framework. Then we will analyze some of the most common SA strategies starting from Pairs Trading (with Stochastic Approach and Cointegration Approach). We will move then to High Frequency strategies which represent one of the most popular set of techniques in today highly volatile markets. Finally we will review some modules trying to exploit inefficiencies related to behavioral phenomena. In the last part of our analysis we will apply SA to 130/30 products using synthetic index swaps with up to date market spreads.

Extrait

Une des idées importantes dans la finance c'est l'absence d'arbitrage, où une pure opportunité d'arbitrage (POA) est définie comme une stratégie de trading a cout zéro qui donne la possibilité d'un gain sans possibilité de perte. L'absence d'arbitrage regarde l'hypothèse d'un marché efficace (HME), en relation auxquels les prix d'une activité reflètent pleinement toute l'information disponible et, dans un marché idéalisé, où il n'y a pas de frictions et couts de trading, peuvent etre produits avec un trading fondé sur l'information parce-que les gains ont déjà eté pris. Nous montrons et testons les principaux défiers à la HME, surtout en relation à la behavioral finance theory, nous étendrons l'idée d'arbitrage identifié statistical arbitrage (SA) comme un des leurs. Un grand nombre des études empiriques sur le S&P 500 conclude que les séries historiques financieres semblent contredire l'efficient market hypothesis. Differéntes potentielles stratégies SA peuvent etre exploitées par des investisseurs rationnels. Nous les décrivons, concentrés surtout sur pairs trading, behavioral statistical arbitrage. De plus, nous testons leur effectif profil SA sur le S&P 500 money center banks sector, relativement à HFR equity market neutral index. Beaucoup de stratégies montrent leurs robustesse out of sample basis. Des effectifs stratégies SA peuvent etre de gestion active des actions. Dans notre recherche, nous étudions des applications de cas réels, sur le S&P 500 money center banks sector, dans une de plus zones novatrices du asset management : articles 130/30. Ces articles sont un type de mutual fund qui permettent aux portfolio managers de tenir positions qu'elles soient

longues (jusqu’à 130%) où courtes (jusqu'à -30%) sur différentes actions. Toutes les stratégies ne sont pas appliquées avec le physical shortening, mais avec les equity swaps, un instrument qui permet de prendre des positions longues et courtes en échange liés au notionnel du swap. Les tests out of sample, relatifs au MSCI USA index, montrent que des effectives stratégies SA améliorent le profil risk- return de gestion active des actions.

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CHAPTER 1- THEORY OF ARBITRAGE

1.0 Introduction

In this chapter we introduce the concept of Statistical Arbitrage (SA), a trading strategy designed to exploit market anomalies, see Hogan et al (2004).

As SA circumvents the joint hypothesis dilemma of traditional market efficiency, we need first to review the Efficient Market Hypothesis (EMH) in its different forms (see Samuelson, 1965, Fama, 1963, 1965a, 1965b and 1970, and Lucas, 1978) with a brief review of the empirical results in its support (see Lo, 2007). In this contest we show the main challenges to the EMH mainly in relation to Behavioral Finance Theory, see Shiller (1981) Thaler (1993) and Lo (2007).

In the second part of the chapter we introduce the concept of SA according to Hogan et al (2004). We will review the various attempts undertaken at extending the concept of arbitrage identifying SA as one of them.

1.1 Efficient Market Hypothesis

The Efficient Market Hypothesis (EMH) is one of the key concepts in the academic literature since the 1960s. According to the EMH, security prices fully reflect all available information. That means that in an idealized market (where there are no frictions and trading costs) no profits can be generated from information based trading as such profits must have already been exploited. In mathematical terms: prices follow martingales.

The EMH was introduced independently by Paul A. Samuelson and Eugene F. Fama in the 1960s. Starting from studies of prices dynamic, Samuelson (1965) claims that, in efficient markets, prices are properly anticipated and move randomly. The analysis of Fama (1963; 1965a; 1965b; 1970) moves instead from the analysis of statistical properties of stock prices and states that efficient markets discount all available information. After the first pioneering studies, EMH evolved to be historically subdivided in three categories (weak, semi- strong and strong), each dealing with a different type of information:

Weak EMH argues it is no possible to gain extra- return by yield time series analysis, because every market discounts immediately this information. This degree of efficiency can be tested by a regression between market returns, at time t, and market returns, at time $t-1$, on a sample period meaningfully long. If there is a significant relationship, the market is weak inefficient, as future returns can be forecasted through past returns.

Semi-strong EMH states that it is not possible to gain extra- return by exploiting public available information, because every market discounts, immediately and precisely, it. To check this market property, several strategies can be tested such as *winners-losers* (i.e. buy over- performing and sell under- performing), *weekly effect* (i.e. buy on Monday and sell on Friday), *January effect* (i.e. buy on 31st December and sell on 2nd January), over a meaningfully long time horizon.

Finally, *Strong EMH* claims it is not feasible to earn extra- returns, by using private information, because every market fully discounts it. To break down this degree of efficiency, it can be studied whether specialized funds beat the market using, for example, the Jensen alpha estimation, by regressing fund returns on market returns. Alphas statistically significantly greater than zero will support market strong efficiency.

A decade after Samuelson's and Fama's landmark papers, the efficient market hypothesis was introduced in to the Neoclassical Theory by Lucas (1978). The economist states that the efficient market is a market where price, weighted for a marginal utility function, moves as a martingale.

1.1.1 The White Noise Hypothesis

If markets are weak efficient then price changes are unpredictable and can be modeled as white noise. In literature, there are several strategies available to check it.

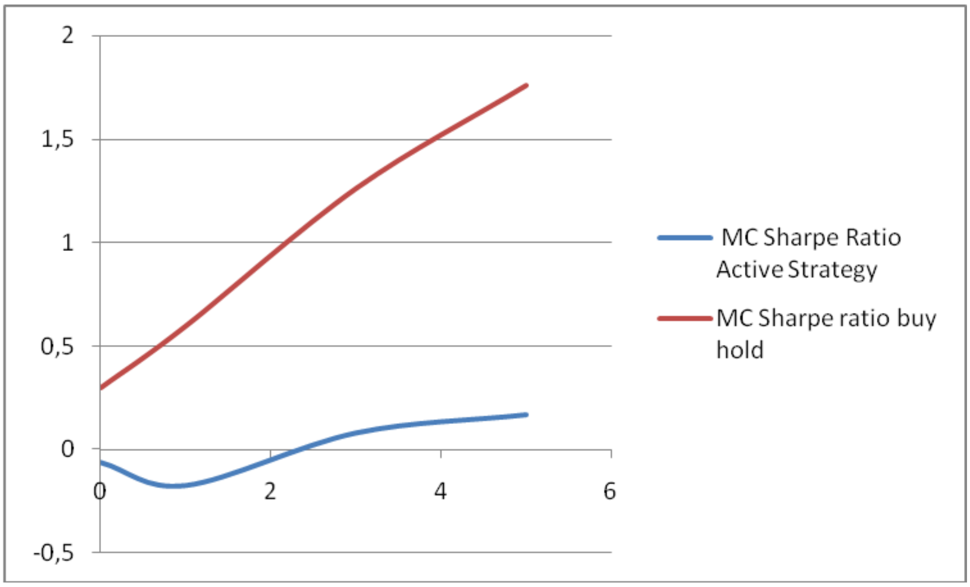
A first test is given by the comparison of the Sharpe Ratio of a *buy-hold* strategy and that of an active strategy. As a naïve active strategy, we can consider buying and holding the S&P 500 for one year, if the return of the S&P 500 in the previous year was positive (or selling it if the return was negative). If the active strategy defeats the buy and hold one, that can mean that the series of the returns has a predictive capacity, and can be modeled as time series.

Consider the S&P 500 daily time series.

Sample period: 01/01/1990-01/01/2003, table:

Strategy	Return	Volatility	Sharpe Ratio
Active Strategy	5.11%	20.11%	-0.06
Buy-hold	11.46%	17.24%	0.30

Plot also 10000 Monte Carlo simulations of Sharpe ratio at 1, 3 and 5 years, both of active and buy-hold strategy



This would support the no, historical returns- based, strategy extra returns. Hence, markets should be no time series modeled and weak efficient.

Another important feature of the white noise is constant variance. In order to analyze it, we can consider the ratio given by

$$VR(t) = \frac{VARIANCE\,(X_K)/K}{VARIANCE\,(X_1)},$$

where $VARIANCE(X_K)$ is the process variance on k-periods and $VARIANCE(X_1)$ is the process variance on one lag.

Ratio movement around 1, on linear scale, or 0, on logarithmic scale, means white noise behavior, no predictability and efficiency.

We test the Variance ratio on the S&P 500.

Key assumptions, data and graphical output:

Short variance: 30 bars

Long variance: 200 bars

Sample period: 01/01/1990- 01/03/2003

Linear scale:

Data

Mean: 1.296

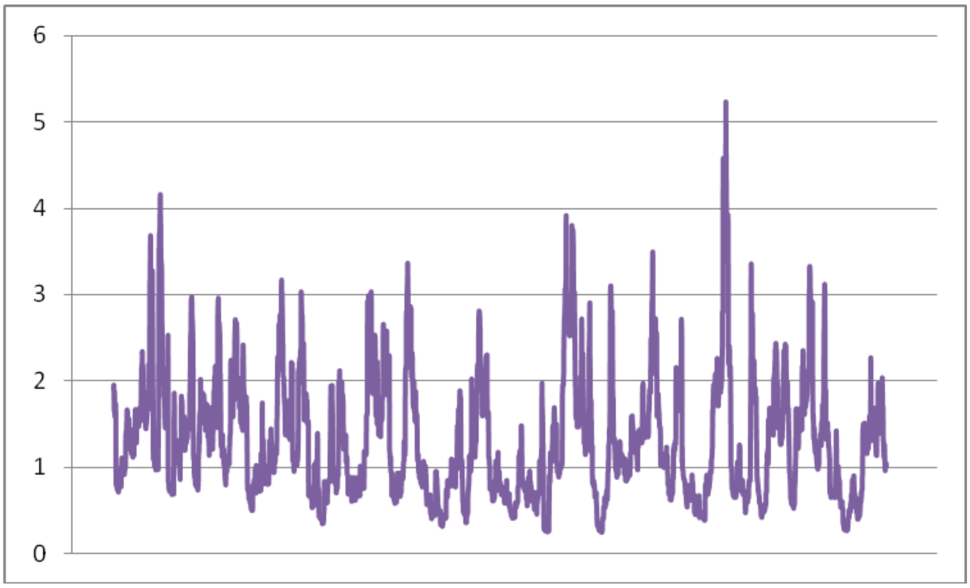
Median: 1.143

Volatility: 0.712

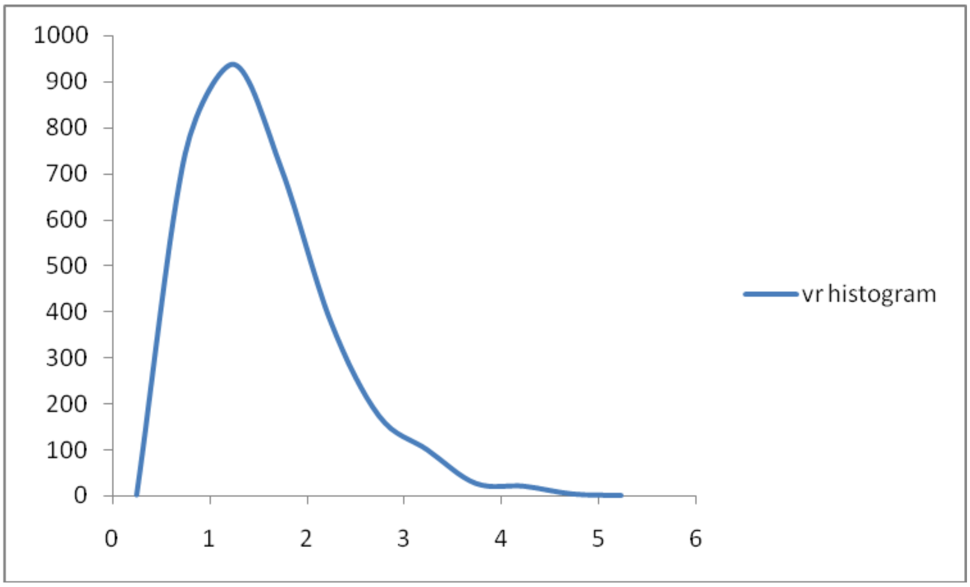
Min: 0.240

Max: 5.223

Time-VR graph



Histogram



Logaritmic scale: data, time graph and histogram

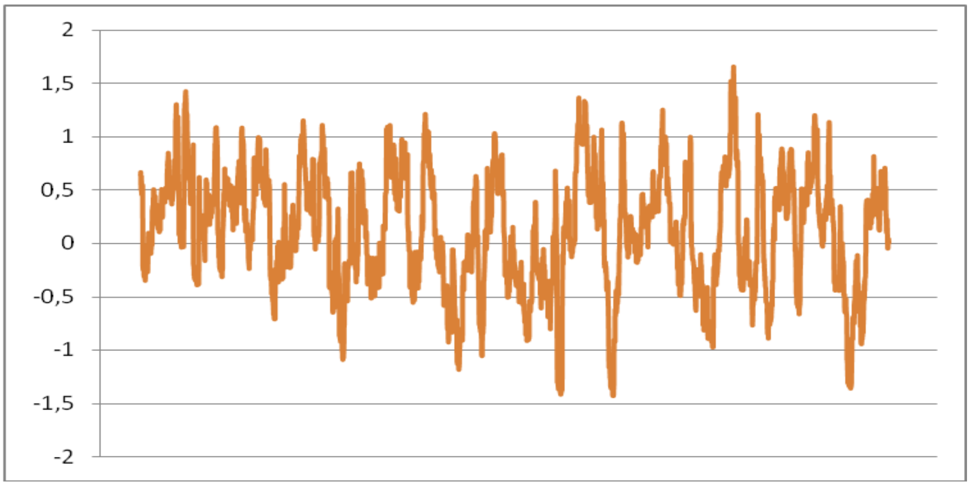
Mean: 0.110

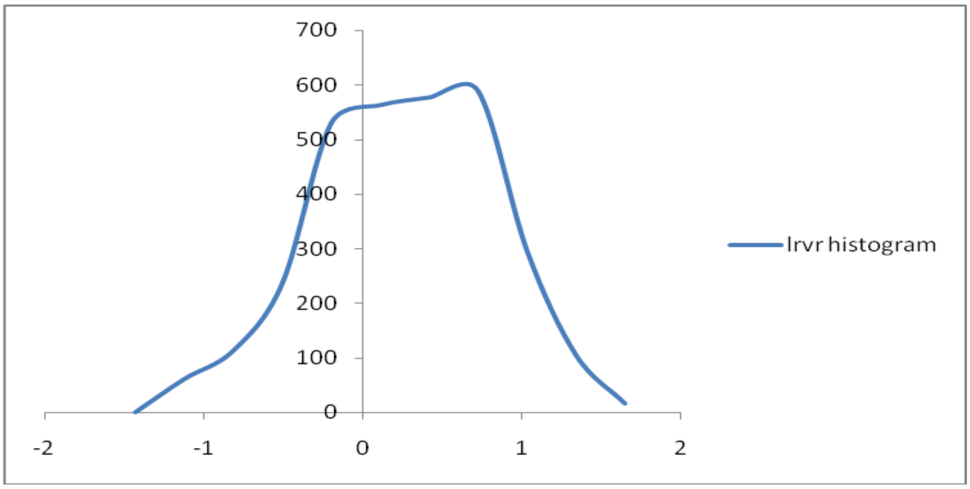
Volatility: 0.559

Median: 0.134

Min: -1.428

Max: 1.653





This would support no random walk and weak inefficient hypothesis.

1.1.2 The Dividend Discount Model Hypothesis

According to the EMH, in a world without uncertainty the market price of a stock must equal the present value of all future dividends. This concept was generalized by Grossman and Shiller (1981), who stated that, in a world of uncertainty, the market prices equal the conditional expectation of the present value of all future dividends, discounted at the appropriate risk-adjusted cost of capital.

In order to test this hypothesis, we can compare the Sharpe Ratio of a Value strategy with that of the S&P 500. A Value strategy consists in investing in undervalued stocks.

We show our test results on a S&P value portfolio.

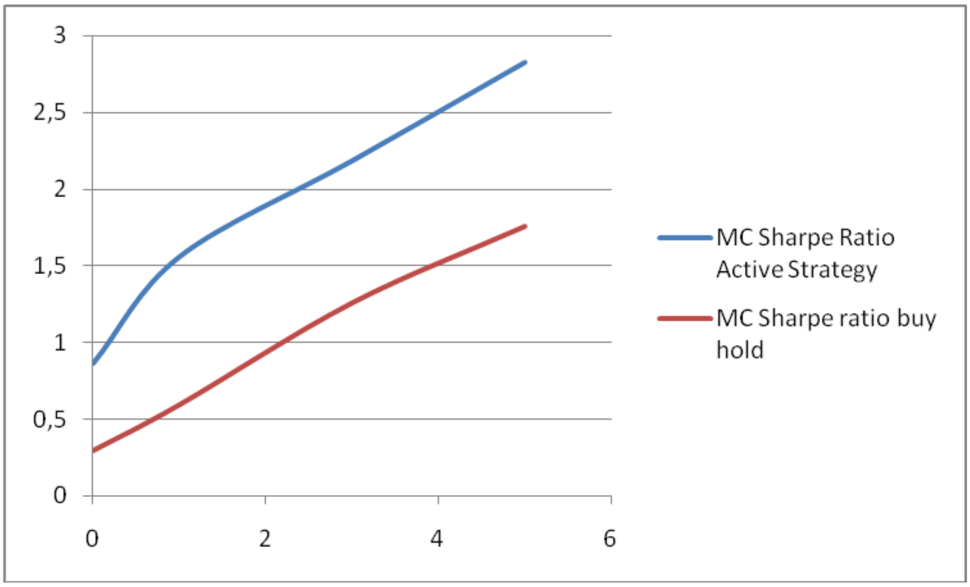
Key data:

Sample period: 01/01/1990- 01/03/2003

Table:

Strategy	Return	Volatility	Sharpe Ratio
Buy-hold	11.46%	17.24%	0.3
Value Strategy	18.76%	14.84%	0.93

Plot also 10000 Monte Carlo simulations of Sharpe ratio at 1, 3 and 5 years, both of Active strategy and buy-hold strategy



We conclude potential inefficiency relative to this hypothesis.

1.1.3 Behavioral Finance

Behavioral Finance is a branch of finance using models in which agents are not fully rational, either because of preferences, or because of mistaken believes.

Among the main behavioral explanations there are:

1) overreaction (DeBondt and Thaler, 1985)
2) underreaction (Ball and Brown, 1968; Bernard and Thomas, 1990)
3) anomalies (Rozeff and Kinney, 1976; Keim, 1983; Roll 1983)
4) risk profits aversion-risk losses seeking (Kanheman and Tversky, 1979; Shefrin and Statman, 1985; Odean, 1998)
5) miscalibration of probabilities (Lichtenstein, Fischoff and Phillips, 1982)

This theory challenges EMH by affirming that investors are irrational and market prices consistently follow predictable patterns of which it is possible to take advantage.

Overreaction. Overreaction is a theory claiming that investors tend to overreact to the market performance. In basic terms, they sell when market is bearish, and buy when it is bullish. This attitude pushes prices far from their fair values, also for a long time. As a consequence patterns are predictable and trend strategies should beat a *buy-hold* strategy. A possible test of Overreaction is given by the comparison of AMA (arithmetic moving average) 200 Sharpe ratio and buy-hold Sharpe ratio strategies, on the S&P 500.

We show our test results.

Key data:

Sample period: 01/01/1990-01/03/2003

Buy hold Strategy:

Return: 11.46%

Volatility: 17.24%

Sharpe ratio: 0.3

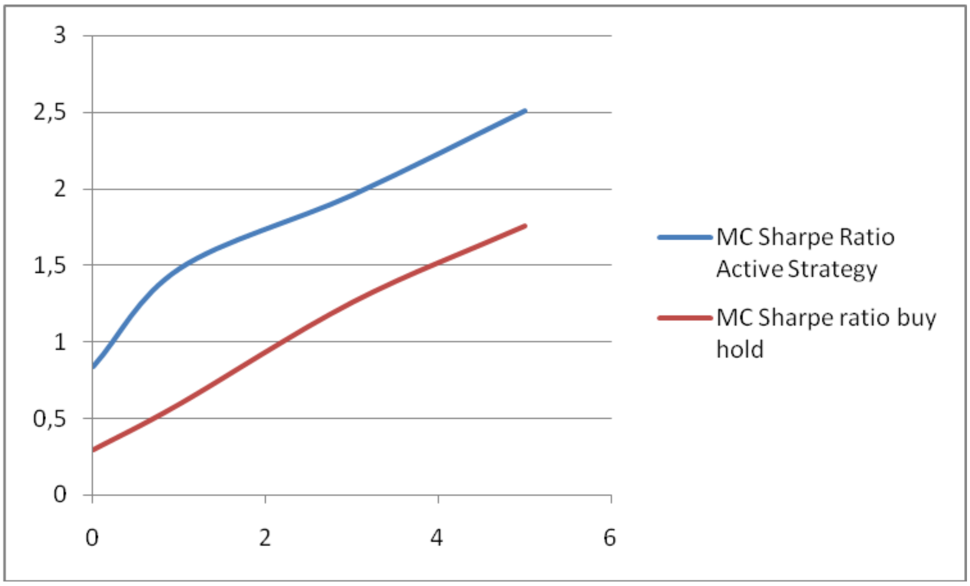
AMA 200 Strategy:

Return: 20.57%

Volatility: 16.69%

Sharpe ratio: 0.84

Plot also 10000 Monte Carlo simulations of Sharpe ratio at 1, 3 and 5 years, both of active and buy-hold strategy



We conclude potential inefficiency relative to this hypothesis.

Underreaction. There is Underreaction when investors underestimate market information, such as earning announcements. A consequence of this is that market prices exhibit predictable drifts. A test for Underreaction can be the Sharpe ratio comparison between a buy-hold strategy and an active strategy, where those securities with higher than expected yearly earnings are bought and held for a pre-specified time horizon. If the active strategy is more profitable than the buy- hold strategy, then this supports the presence of Underreaction.

Test results.

Key data:

Sample period: 01/01/1990-01/03/2003

Buy hold Strategy:

Return: 11.46%

Volatility: 17.24%

Sharpe ratio: 0.3

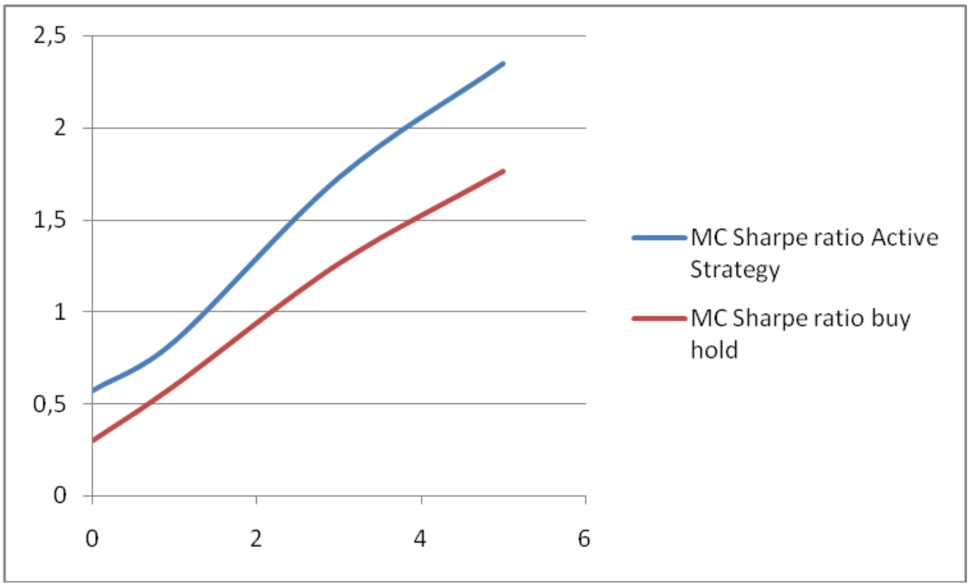
Underreaction Strategy:

Return: 18.57%

Volatility: 21.18%

Sharpe ratio: 0.56

10000 Monte Carlo simulations of Sharpe ratio, at 1, 3 and 5 years, both of active and buy-hold strategy



We conclude potential inefficiency relative to this hypothesis.

Anomalies. In the context of EMH theory researchers refer to anomalies as regular patterns in assets' returns which are reliable, widely known and inexplicable. Among the most popular anomalies there is the so called January Effect. In order to detect the presence of January Effect we can compare the Sharpe ratio of a buy-hold strategy and an active strategy by which securities are bought on the last trading day of the year at the close and sold at the open of the first trading day of the year.
We show our test results.

Key data:

Sample period: 01/01/1990-01/03/2003

Buy-hold Strategy:

Return: 11.46%

Volatility: 17.24%

Sharpe ratio: 0.3

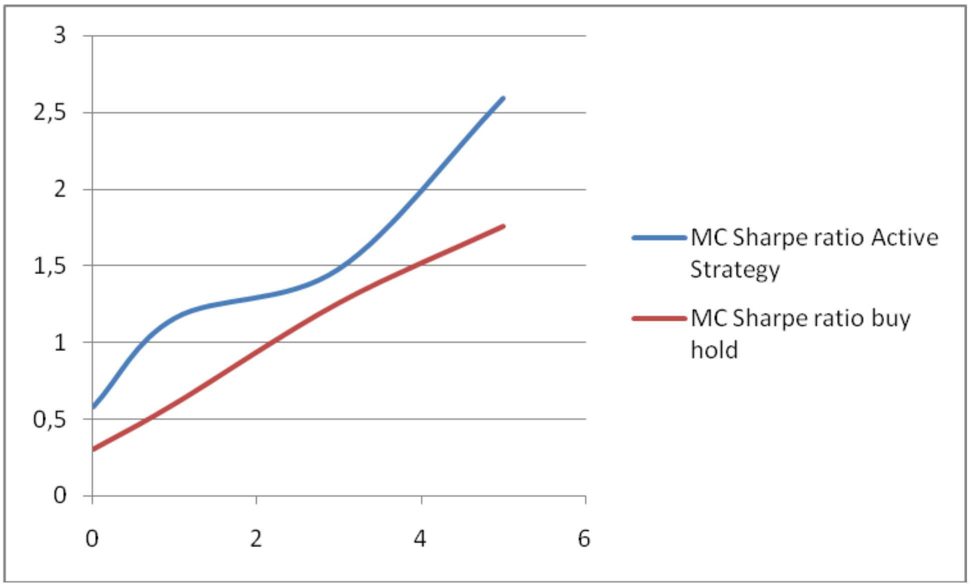
January Effect Strategy:

Return: 20.02%

Volatility: 23.75%

Sharpe ratio: 0.58

10000 Monte Carlo simulations of Sharpe ratio at 1, 3 and 5 years, both of active and buy-hold strategy



We can conclude that the January Effect on the S&P 500 exists.

Risk Aversion - Risk Seeking Imbalance. According to behaviouralists, quantitative models, based on market efficiency, are likely to be wrong, as most individuals tend to select wrong answers. An empirical evidence could be the experiment conducted by two psychologists, Kahneman and Tversky (1979). Suppose an individual is given the opportunity to choose among the investment opportunities A or B, where

1) A yields 240 Euro
2) B yields 1000 Euro at 25 per cent and 0 Euro at 75 per cent

Kahneman and Tversky report as most subjects prefer A. Now let us suppose that the same individuals are offered the opportunity of choosing among investments C and D where

1) C yields a loss of 750 Euro
2) D yields a loss of the 1000 Euro at 75 per cent and 0 Euro at 25 per cent

In this case most subjects will choose D. Clearly there is no right or wrong choice.

But let consider the combination of choices A and D has the pay-off

1) A+D: -760 Euro at 75 per cent
2) A+D: 240 Euro at 25 per cent

However, B and C investment pays- back

1) B+C: -750 Euro at 75 per cent
2) B+C: 250 Euro at 25 per cent

Hence, B and C gain a risk- free 10 Euro profit. On the back of that, behaviouralists conclude that investors tend to chose irrationally.

Irrational Probability Beliefs. Supporters of EMH argue there is in any case a limit to the impact of irrational probability beliefs. An example is given by the so called Dutch Book (at study of which contributed the Italian Mathematician De Finetti). Consider an investor with the following beliefs:

1) S&P 500 falling- probability 50 per cent
2) S&P 500 rising- probability 75 per cent

Let consider the following two bets:

1) Bet 1: $1 if the S&P 500 falls and -$1 otherwise
2) Bet 2: $1 if the S&P 500 rises and -$3 otherwise

If an investor can place $50 on the Bet 1 and $25 on the reverse of Bet 2, then, if the S&P 500 rises, the investor will lose $ 50 from Bet 1 but will earn $ 75 from Bet 2. In case the S&P 500 falls, the investor will earn $50 from Bet 1 and loose $ 25 from Bet 2. Regardless of the outcome, the strategy will yield a riskless profit of $25.

1.2 Statistical Arbitrage

1.2.1 Brief History

It is generally accepted that Statistical Arbitrage (SA) took its first steps in the mid-80's with Nunzio Tartaglia who assembled at Morgan Stanley a team of physicists, mathematicians and computer scientists to uncover statistical mispricings in the equity markets. Tartaglia's group of former academics used statistical methods to develop trading programs, executable through automated trading systems, which replaced traders' intuitions and skills with disciplined, consistent filter rules, see Gatev et al (2006). One of the reasons which made SA particularly appealing to quantitative analysts is that while absolute pricing is a notoriously difficult process with a wide margin of error, relative pricing can be slightly easier. Relative pricing means that two securities that are close substitutes for each other should sell for the same price: it does not say what that price should be.

SA fortunes turned at the end of the 90's. The most striking example is the LTCM, an hedge fund founded in 1994 in which worked the Nobel Prize winners Sholes and Merton. The company had developed complex statistical arbitrage models which were initially enormously successful. Anyway, in 1998, on the back of the financial crises in East Asia and Russia, the LTCM's arbitrage strategies started producing huge losses which almost left the markets in ruins and forced the Federal Reserve Bank of New York to organize a bail out in order to avoid a wider financial collapse. Among the SA strategies played by LTCM, few were as widely used as Fixed Income arbitrage which was one of the main causes of losses for almost every major investment banking firm in Wall Street. Nevertheless, SA techniques kept on growing in popularity till 2008, making their entrance into the world of mutual funds (traditionally long only investments) mainly with the very recent introduction (in Ireland in spring 2008) of 130/30 products, a type of mutual fund allowing asset managers to hold both long (up to 130%) and short (up to -30%) positions on different securities. Furthermore, it has to be noted that, paradoxically, the recent restrictions in short selling (September 2008) and the need for stricter regulations could be beneficial for 130/30 products, which could take advantage from the growing disaffection for the largely unregulated hedge funds and prompt a new phase in the history of SA.

Surprisingly, in spite of the widely believed inefficiency of commodity markets and their impressively growing importance, Riddley (2006) reports that still in 2004 there were very few arbitrageurs in commodities. Riddley suggests that this is due to the fact that the insufficient capital dedicated to make those markets efficient, on the one hand contributed to the number of profit-making opportunities but on the other hand meant that anomalies can often linger or worsen, generating uncertainty for arbitrageurs and increasing their chances of losses. It is in this context that our research aims to uncover potential opportunities but even though we will not be able to find any of them it will always represent a novel attempt to extensively use SA strategies across the different commodities.

1.2.2 Arbitrage and its Generalizations

One of the key concepts in finance is the absence of arbitrage, where a pure arbitrage opportunity (PAO) is defined as a zero cost trading strategy that offers the possibility of a gain with no possibility of a loss. As a large number of empirical studies conclude that financial time series appear to contradict the efficient market hypothesis (and so the absence of PAO), various attempts at extending the concept of arbitrage have been undertaken. In the context of incomplete markets, Cochrane and Saa-Requejo (2000) invoke Sharpe ratios to find asset prices that are good deals for investors, while Bernardo and Ledoit (2000) exclude investments whose maximum gain-loss ratios are too attractive (they call them approximate arbitrage opportunities, AAOs). Both of these approaches investigate trading opportunities that generalize the definition of arbitrage by specifying a particular model of market equilibrium (as suggested by Fama in 1998). Carr, Geman and Madam (2001) introduce the concept of acceptable opportunities which would be executed by any reasonable investor, implying their existence is incompatible with an efficient market.

1.2.3 SA Definitions and Theoretical Framework

Statistical Arbitrage (SA) represents another attempt at arbitrage generalization but, in spite of its popularity, there is no universally accepted definition. According to Pole (2007) the term SA was first used in the early 90's and several empirical definitions can be found, see for example Thomaidis/Kondakis (2006). A common point for many researchers is that SA is defined under the observed statistical measure rather than a collection of probability measures. A more rigorous definition can be found in Hogan et al (2003) where, paraphrasing the formulas, SA is defined as a long horizon trading opportunity that generates risk-less profit. As such, SA is a natural extension of the trading strategies utilized in the existing empirical literature on anomalies and can be seen as the time series analogue of the limiting arbitrage opportunity (LAO) contained in Ross (1976). Bondarenko (2002) gives another definition of SA introducing the concept of Statistical Arbitrage Opportunity (SAO) as a zero cost trading strategy for which the expected payoff is positive and the conditional expected payoff in each final state is not negative. For a more detailed review please refer to Lazzarino (2008).

1.2.4 Statistical Arbitrage and Market Efficiency

This paragraph introduces the concept of SA according to Hogan et al (2003). They define SA as a long horizon trading opportunity that generates riskless profits using only publicly available information (for instance, past returns, firm sizes, earnings announcements, market versus book values, sales growth rates and macroeconomics conditions). Before giving the definition we need some notation. Let consider a strategy which trades $x(0)$ units of a stock S_t and $y(0)$ units of a risk-free asset B_t, with initial value given by $B_0 = 1$. Let suppose $V(t)$ is the set of the cumulative profits of the strategy given by $x(t)S_t + y(t)B_t$ with discounted cumulative profits $v(t) = V(t)/B_t$.

Definition (Hogan et al ,2003) A SA trading strategy is a zero cost, self financing trading strategy $(x(t): t \geq 0)$, with a cumulative discounted value $v(t)$ such that

(i) $v(0) = 0$

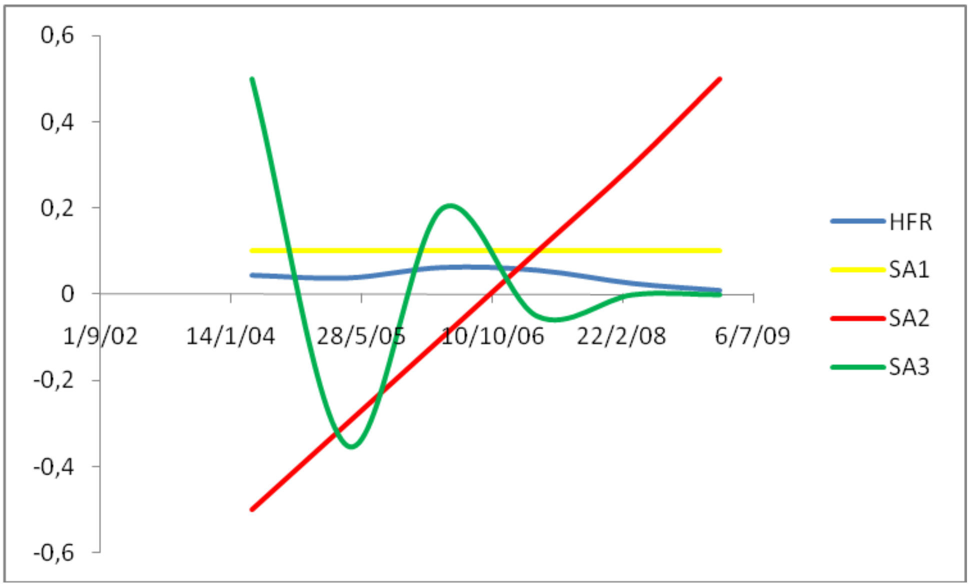
(ii) $\lim\limits_{t \to \infty} = E^P[v(t)] > 0$

(iii) $\lim\limits_{t \to \infty} P(v(t) < 0) = 0$

(iv) $\lim\limits_{t \to \infty} \dfrac{Var^P[v(t)]}{t} = 0$ if $P(v(t) < 0) > 0, \forall t < \infty$

By condition (i) SA is a zero cost trading strategy with (ii) positive expected discounted profits and (iii) a probability of loss converging to zero. Condition (iv) requires that the time-averaged variance converges to zero if there is always a positive probability of loss. It is a statistical requirements allowing variance to grow on time, but less than linearly.

Graphically, holding period-return of SA strategies can be as below:



Definition: A standard arbitrage is a trading opportunity **for which**

(i) $V(0) = 0$

(ii) $\exists T > 0$ s.t. $\forall t \geq T$

- $P[V(t) > 0] > 0$ and
- $P[V(t) \geq 0] = 1$

Note 1: It is immediate to verify that a standard arbitrage is a special case of SA.

Note 2: According to Hogan, this definition is similar to the limiting arbitrage opportunity of the APT Model (see Ross, 1976). The difference between the two concepts is that the former is a time strategy, while the latter is a cross section strategy at a point in time. Hence, APT can be exploited in a market that stems many assets and Statistical Arbitrage can be exploited for long time horizons in every kind of market.

A basic comparison of SA with standard arbitrage reveals that $\exists T > 0$ such that $P[v(T) < 0] < \varepsilon$ for SA while for a standard arbitrage that would be $P[v(T) < 0] = 0$. Thus comparing the two definitions it appears that SA and standard arbitrage are only separated by an ε probability of a loss.

CHAPTER 2- SA TRADING STRATEGIES

2.0 Introduction

In this chapter we will introduce some of the most important SA strategies in equities. We will focus mostly on those strategies which are more popular at the moment, in the attempt of producing an updated review on some of the topical methods for quantitative investment, in the current markets environment.

We will start by introducing the concept of Pairs Trading. After a brief history, we will discuss some Pairs Trading cutting hedge applications used in today's markets, with particular emphasis on the Stochastic Spread approach and Cointegration approach. In the central part of the chapter we will review some High Frequency techniques with emphasis on computational efficiency and erratic data handling. At last we will move to Behavioral strategies with reference to Contrarian and Value strategies.

2.1 Pairs Trading

2.1.1 Introduction and Brief History

Pairs Trading is a popular and well established trading strategy whose concept is disarmingly simple. "Find two stocks whose prices have historically moved together, when the spread between the two widens, short the winner and buy the loser: if history repeats itself, prices will converge and the arbitrageur will profit" (Pole, 2007). At first, Pairs Trading can sound disarmingly simple but it can be extremely challenging to put into practice.

Pairs Trading took its first steps in the mid-80's and, since then, it has been widely adopted by traders and hedge funds with mixed fortunes. Historically, Pairs Trading strategies relied on either fundamental valuation or technical analysis. The use of fundamental valuations requires extensive research on individual companies to identify pairs that are potentially driven by similar economic factors; however it heavily relies on personal judgment of financial analysts. Technical analysis relies instead on the use of indicators such as Bollinger Bands or Relative Strength; however the effectiveness of these methodologies has often yielded mixed results.

More recently, interest in Pairs Trading has resurfaced but with a distinctly different approach to the problem, relying on a vast array of statistical techniques. Owing to the proprietary nature of the strategies, published research that is directly related to the topic is quite limited. However it is possible to identify two main Pairs Trading approaches: Stochastic Spread approach and Cointegration approach.

2.1.2 Pairs Trading: Strategy Description

Pairs Trading is a trading strategy which aims to exploit temporal deviations from an equilibrium price relationship between two securities. It is given by a long position in one security and a short position in another security in such a way that the resulting portfolio is market neutral (which typically translates in having a beta equal to zero). This portfolio is often called a spread. Pairs Trading involves putting on positions when the spread is substantially away from its mean value with the expectation that the spread will revert back. It requires defining spread models and trading rules.

- identifying pairs of stocks
- calculating the optimal ratio between the two securities in the pair
- defining Entry and Exit Levels

Spread Modeling Among the more recent techniques we will review the Stochastic Spread approach and the Cointegration approach.

According to the Stochastic spread approach, a generic spread between two securities is modeled as a Vasicek stochastic process. This is quite an innovative technique which relies mostly on the model capability to explain the observed phenomenon.

The Cointegration approach instead uses cointegration as a mean to capture time series dependency. It is probably the most popular approach in quantitative and hedge funds and a reasonable amount of literature has been spent on it.

Both the approaches will be discussed in the following paragraphs.

Trading Rules. A vast array of trading rules is available to practitioners. Here we will just briefly report the fundamental points as a more detailed discussion goes beyond the purposes of the present analysis.

Identifying Stock Pairs. Properly identifying pairs represents one of the key issues. Some quantitative techniques are available mostly based on stocks distance measurement. As this leaves the door open to spurious analysis, a more traditional qualitative judgment quite often is associated.

Pair Ratio. Once an appropriate pairs of stocks has been selected, it is necessary to identity the strategy securities weights, the so called Pairs Ratio. This ratio needs to meet two requirements. First of all, it should make the strategy zero cost. Finally, it needs to make the strategy market neutral.

Entry Level. To open a strategy it is necessary to identify when a pair significantly differs from the fair value identified through spread modeling techniques. In defining Entry and Open levels, volatility measures play a major role. Basic rules open positions when the securities ratio is a multiple of standard deviations away from the mean (to close them when the ratio returns to mean). More sophisticated approaches are available even though this is one of the areas in which the propriety nature of the strategies makes finding literature and documentation particularly difficult.

Exit Level. They are necessary to close a position and can be classified in Take Profit and Stop Loss. Take Profits close a strategy in profit when the securities ratio goes back to model "fair" values. Take Profit shares several similarities with Entry levels. A slightly different reasoning applies to Stop Loss. A Stop Loss applies when the strategy outcomes diverge from expectations and the cumulative losses exceed levels which can be determined through a variety of techniques and mostly refer to the maximum affordable value at risk.

2.2 Stochastic Spread Approach

According to the stochastic approach spreads are modeled with Vasicek stochastic processes. The use of a Vasicek mean reversion process makes Pairs Trading an SA strategy. To implement the strategy, spread modeling needs to be coupled with filtering and estimation techniques. Given the sophistication of the approach, there is little empirical work based on it with quite a dearth of reported trading rules associated with this strategy.

2.2.1 The State Process

Stochastic Spread Approach (Elliot et al, 2005). In a given probability space (Ω, F, P), consider a state process $\{x_k \mid k = 0,1,2,...\}$ with $x_k \in \Re$ representing the Pairs Trading spread at time $t_k = k\tau$ where τ is the time lag. We assume that $\{x_k\}$ is mean reverting:

$$\begin{cases} x_{k+1} - x_k = (a - bx_k)\tau + \sigma\sqrt{\tau}\varepsilon_{k+1} \\ a,b,\sigma,\tau \in \Re \\ b,\tau > 0 \\ \sigma \geq 0 \\ \{\varepsilon_k\} \sim N(0,1), iid \end{cases} \qquad (1)$$

and ε_{k+1} *independent of* $x_0, x_1, ..., x_k$

Note 1 The process reverts to $\mu = a/b$ with strength b

Note 2 The process x_k is so characterized

$$\begin{cases} x_k \sim N(\mu_k, \sigma_k^2) \\ \mu_k = \dfrac{a}{b} + \left[\mu_0 - \dfrac{a}{b}\right](1 - b\tau)^k \\ \sigma_k^2 = \dfrac{\sigma^2\tau}{1-(1-b\tau)^2}\left[1-(1-b\tau)^{2k}\right] + \sigma_0^2(1-b\tau)^{2k} \end{cases}$$

Note 3 If we consider $|1 - b\tau| < 1$ it is immediate to see that

$$\mu_k \to \frac{a}{b} \text{ as } k \to \infty$$

$$\sigma_k^2 \to \frac{\sigma^2 \tau}{1 - (1 - b\tau)^2} \text{ as } k \to \infty$$

Note 4 It is possible to write (1) as

$$\begin{cases} x_{k+1} = A + Bx_k + C\varepsilon_{k+1} \\ A = a\tau \geq 0 \\ a < B = 1 - b\tau < 1 \\ C = \sigma\sqrt{\tau} \end{cases} \tag{2}$$

This will be the formulation of the spread we will refer to from now onwards.

2.2.2 The Observation Process

If we allow the presence of a measurement noise $\{\omega_k\}$, we can define the observation process $\{y_k\}$ as

$$\begin{cases} y_k = x_k + D\omega_k \\ D \in \Re, D > 0 \\ \{\omega_k\} \sim N(0,1), iid \end{cases} \tag{3}$$

where $\{\omega_k\}$ are independent of $\{\varepsilon_k\}$.

For a given set on information $Y_k = \sigma\{y_0,...,y_k\}$ from observing $y_1,...y_k$ the strategy target is given by calculating

$$\hat{x}_k = E[x_k \mid Y_k] \tag{4}$$

which requires estimating (A,B,C,D). The equation (4) enable us to say that if $y_k > \hat{x}_{k|k+1} = E[x_k \mid Y_k]$ the spread is too large and a strategy can be initiated. The opposite holds too.

2.2.3 Stochastic Spread Approach: the Algorithm

From (2), (3) and (4) we obtain that, assuming a Vasicek model for the spread between two securities, $\{x_k\}$ is given by

$$\begin{cases} x_{k+1} = A + Bx_k + C\varepsilon_{k+1} \\ A = a\tau \geq 0 \\ a < B = 1 - b\tau < 1 \\ C = \sigma\sqrt{\tau} \end{cases}$$

with a model for the observed spread $\{y_k\}$ given by

$$\begin{cases} y_k = x_k + D\omega_k \\ D \in \Re, D > 0 \\ \{\omega_k\} \sim N(0,1), iid \end{cases}$$

where $\{\omega_k\}$ are independent of $\{\varepsilon_k\}$.

We need to calculate the conditional expectation (filter)
$\hat{x}_k = E[x_k \mid Y_k]$given $Y_k = \sigma\{y_0,...,y_k\}$. The estimation of the parameters (A,B,C,D) can be realized through the use of different techniques. Elliot et al (2005) highlight how the previous process particularly suits the use of Kalman Filter with EM – Algorithm estimation.

2.2.4 Stochastic Spread Approach: an Application

In this part of our analysis we report a simplified example of application. We will consider the pair Bank of America (BOA) – Citigroup (CITI). We will calculate a spread index given by trading a unit of each security.

The index will be based to 100 starting from 01/03/2003.

The time series considered range from 01/03/2003 till 20/02/2009.

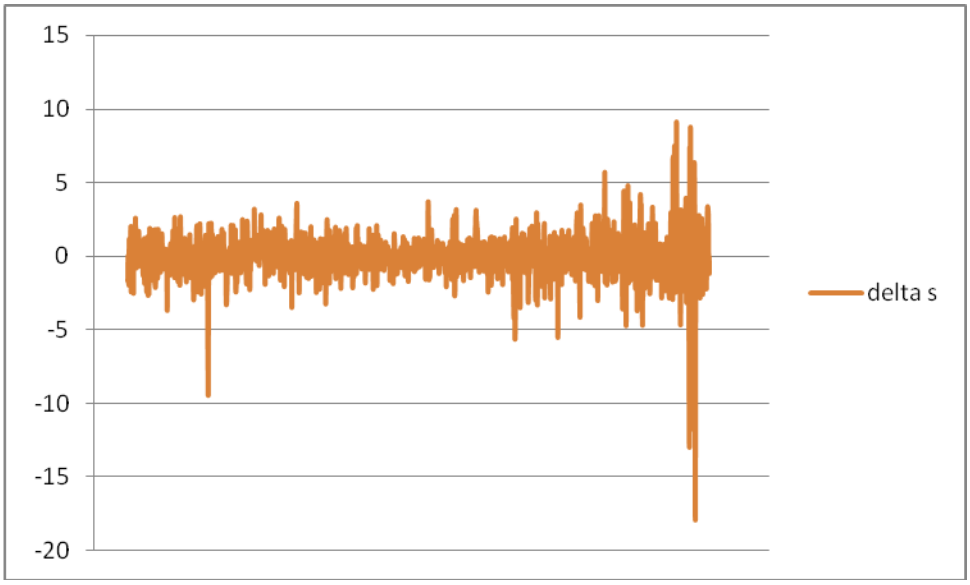
We will model the spread assuming a Vasicek model where D=0 in equation (3), C is known in equation (2) and $x(0) = y(0)$.

A strategy will be opened when the spread exceeds the fair value and closed when it comes back, reverting the trades.

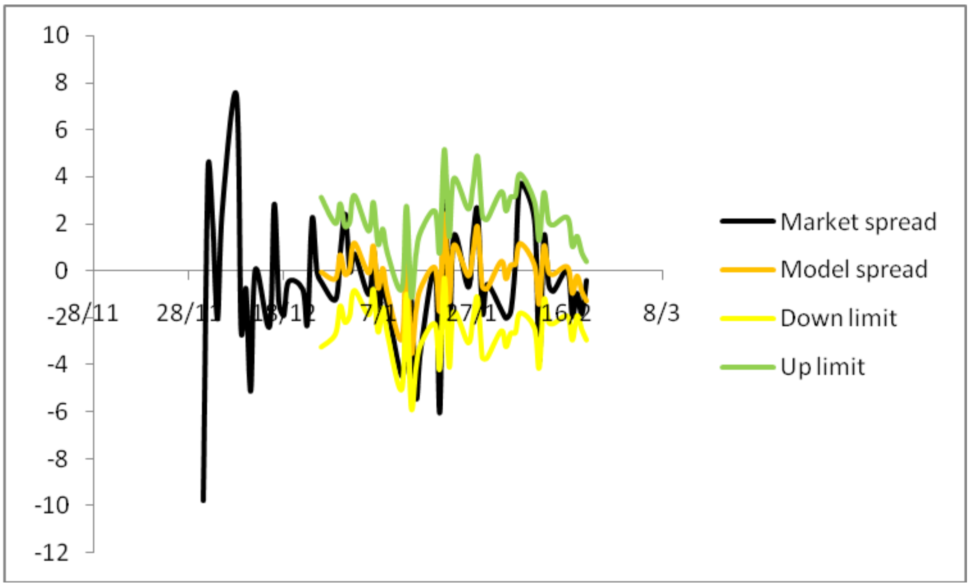
Given the example assumptions, the parameters A and B will be then estimated.

At last, we compare the strategy risk- return profile with the HFR Equity Market Neutral index.

First of all, we have to model the following sample period process



The sample Vasicek difference equation will be $(BOA - CITI)_{t+1} = 0.024 - 0.003 \cdot (BOA - CITI)_t$ where 0.024 is the long run estimated spread variation and 0.003 is the mean- reversion estimated spread speed calculated over the in – sample period.



The out of sample results show that with 8 trades 5 were closed in take profit and 3 in stop loss with the below characteristics

	Return Mean	Return Volatility	Return-Risk Ratio
Stochastic Approach Pairs Trading	6.78%	7.28%	0.92
HFR Equity Market Neutral	3.03%	2.88%	1.05

2.3 Cointegration Approach

With this approach spreads are modeled through the use of cointegration. Engle and Granger (1987) give the formal definition of cointegration among two variables as follows.

Cointegration. Time series Y_t and X_t are said to be cointegrated of order d,b where $d \geq b \geq 0$ written as $Y_t, X_t \sim CI(d,b)$, if (a) both series are integrated of order d, (b) there exists a linear combination of these variables, say $AY_t + BX_t$ which is integrated of order $d-b$. The vector $\{A,B\}$ is called the cointegrating vector.

Note. From now on we will refer to cointegration with $d = b = 1$

The main benefit of cointegration comes from the fact that if we consider two cointegrated time series X_t and Y_t (both of them $I(1)$, so that there is a linear combination of them which is stationary ($I(0)$) then the regression of equation

$$Y_t = A_1 + BX_t + u_t \tag{5}$$

is not spurious and provides the long run relationship between the two variables (the SA equilibrium).

Dickey Fuller Test. Dickey and Fuller (1979, 1981) devised a procedure to test for non-stationary. This is based on the fact that testing for non-stationary is equivalent to testing for the existence of a unit root. The Dickey-Fuller (DF) test is based on three different regressions

$$\Delta Y_t = AY_t + u_t$$
$$\Delta Y_t = A + BY_t + u_t$$
$$\Delta Y_t = A + Bt + CY_t + u_t$$

A variant of DF is given by the Augmented Dickey-Fuller test (ADF), which includes extra lagged terms of the dependent variable in order to eliminate autocorrelation. It is given by

$$\Delta Y_t = AY_t + B\sum_{i=1}^{n} \Delta Y_{t-i} + u_t$$
$$\Delta Y_t = A + BY_t + C\sum_{i=1}^{n} \Delta Y_{t-i} + u_t$$
$$\Delta Y_t = A + Bt + CY_t + D\sum_{i=1}^{n} \Delta Y_{t-i} + u_t$$

Recombining equation (5) it is possible to specify the Error Correction Model (ECM(1,1))

$$\Delta Y_t = A + B\Delta X_t + C(Y_{t-1} - DX_{t-1}) + \varepsilon_t \qquad (6)$$

Which will have is the advantage of including both long-run and short run information. Indeed in this model *B* is the impact multiplier (the short-run effect). It measures the immediate impact that a change in X_t will have on a change in Y_t.

Variance Ratio Test. The Variance Ratio Test detects the process stationarity and it is given by the below ratio

$$VR(t) = \frac{\frac{VARIANCE(X_K)}{K}}{VARIANCE(X_1)},$$

where $VARIANCE(X_K)$ is the process variance on k-periods, k is the period number and $VARIANCE(X_1)$ is the process variance on one lag. If it moves around 1 there is random- walk behavior and no predictability.

2.3.1 Cointegration Approach: the Algorithm

To estimate the cointegration parameters, Engle and Granger proposed the following methodology.

1. **Integration Order Check**. Cointegration requires that the variables are integrated of the same order. The Dickey-Fuller and the Augmented Dickey-Fuller tests can be applied in order to infer the number of unit roots (if any) in each of the variables.
   - If X_t and Y_t are stationary ($I(0)$), it is not necessary to proceed as standard time series methods apply
   - If X_t and Y_t are integrated of different order, it is possible to conclude that they are not integrated

- If X_t and Y_t are integrated of the same order then it is necessary to proceed at step 2

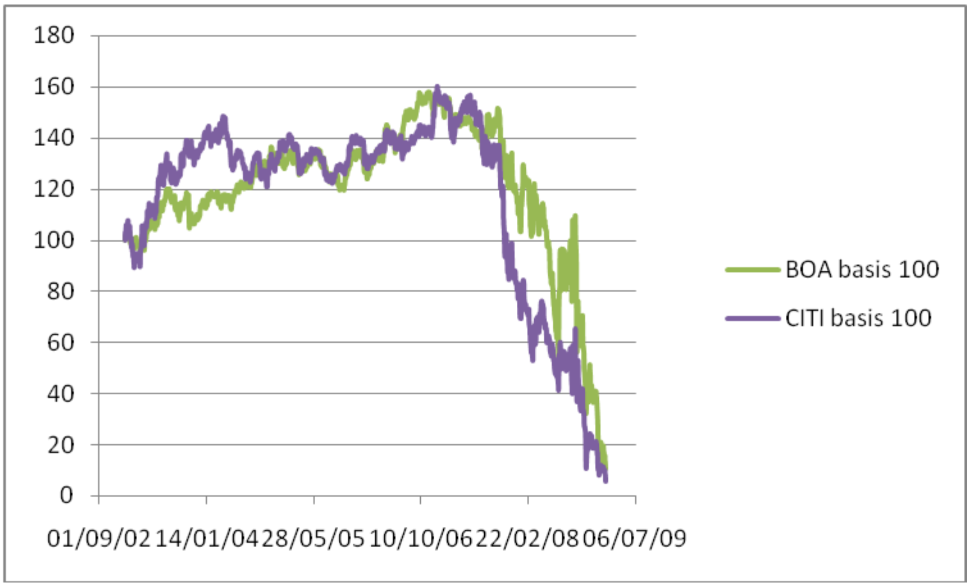
2. **Regression Parameters Estimation**. The next step requires the estimation of the long run equilibrium $Y_t = A + BX_t + e_t$. The parameters estimated with OLS will be useful to assess if X_t and Y_t are really cointegrated.
3. **Check for Cointegration**. A DF test will be applied to the estimated residuals $\hat{e}_t$. If $\hat{e}_t \sim I(0)$ then X_t and Y_t are cointegrated.
4. **Error Correction Model**. If the variables are cointegrated then the ECM can be estimated in order to find the long run and short run effects of the variables.

2.3.2 Cointegration Approach: an Application

Here we report an example of strategy with the use of cointegration.
The trading rules used here will be derived from market practice.
In details we will open a strategy when the ratio hits a 2 standard deviation band from the mean for two consecutive times on 130 days.
The position will be closed when the ratio hits the mean (in Take Profit) or when the strategy loss reaches 2% (in Stop Loss).
The ratio will be fixed in such a way that the spread beta is lower than or equal to 0.2 in order to seek market neutrality.

We will consider the pair Bank of America (BOA)-Citigroup (CITI) over the 01/03/2003-20/02/2009 period. At last, we compare the return-risk profile of this strategy with the HFR Equity Market Neutral index.

First, we show the spread pattern (based 100 on 01/01/2003), between 01/01/2003 and 28/11/2008



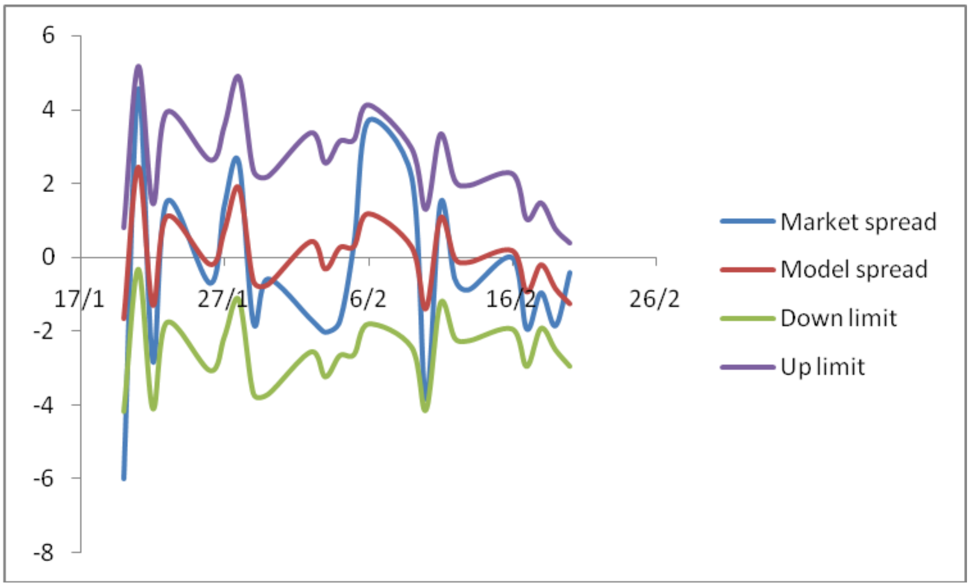
We analyze the cointegtration and then we estimate the ECM(1,1) which will be given by

$$\Delta BOA_t = 0.215 + 0.913\Delta CITI_t - 0.005BOA_{t-1} + 0.003CITI_{t-1}$$

Between 01/12/2008 and 20/02/2009, the cointegration approach based strategy generates 7 trades which carry a positive return with cumulative results summarized in the following table

	Return Mean	Return Volatility	Return-Risk Ratio
Stochastic Approach Pairs Trading	6.94%	4.89%	1.419
HFR Equity Market Neutral	-0.80%	1.61%	-0.632

Below see the graph of the strategy



2.4 High Frequency SA

In this part of the chapter we will focus on strategies specialized in exploiting SA opportunities in a high frequency environment. High frequency strategies are particularly important as mispricing rapidly disappears in highly competitive markets, see Frenkel and Levich (1975, 1977). Successful arbitrage traders need so to be able to rapidly take advantage of market inefficiencies. This appears to be even more important at the light of the fact that most of the studies on arbitrage have mainly been limited to daily data and so might miss many of the available intraday opportunities. Moreover, among the studies which have examined intraday data, many analyzed only a minute fraction of the data because of the need for simultaneous observations.

Studying high frequency financial data set presents additional issues to conventional modeling, mostly related to erratic observations handling and computational efficiency.

Among the proposed methodologies for dealing with erratic data, Muller et al (1990), suggest methods of linear interpolation between erratic observations to obtain a regular homogeneous time series. Bolland and Connor (1996) treat instead erratic arrival as a missing data problem considering heavy tailed distributions.

2.4.1 High Frequency Trading Strategies

We will briefly review now some of the most common high frequency trading techniques.

Technical Analysis. This is probably one of the most common trading tools. Technical analysis is the study of historical trends in market prices in order to predict future price movements. In this area of finance particular importance is given to charts reading where the identification of supports, resistances and trends are the key elements. In order to detect those elements, a large array of indicators (at different levels of complexity) is available. Among the main advantages of technical analysis there are computational efficiency, easy understanding and immediateness of implementation. This is why many trading platforms use it and technical levels are common knowledge among investors.

Robust Kalman Filter. Bolland and Connor (1996) describe how outliers can be robustly identified/filtered in multivariate non-linear data. The formulation of the state problem is given by the classical

$$\begin{cases} x_t = f(x_{t-1}) + e_t \\ z_t = H_t x_t + v_t \end{cases}$$

The state transition vector $f(x_{t-1})$ represents the system dynamics and can be linear or non-linear. H is the observation matrix. The system has two types of noise. The state noise e_t represents the variation due to arbitrage dynamics. The observation noise v_t has two components $v_t = u_t + w_t$ where the first component u_t represents the variation caused by the transaction costs and bid-ask spread. The second component w_t represents the additive outliers within the data. Their methodology allows that on any given second only the rows of the observation matrix which corresponds to an actual observation are used to update the filtering equations. Indeed the underlying states x_t are estimated by a Robust Kalman filter where the predicted state vector $\hat{x}_t$ and the predicted observation $\hat{z}_t$ are given by

$$\begin{cases} \hat{x}_t = f(\tilde{x}_{t-1}) \\ \hat{z}_t = H_t \hat{x}_t \end{cases}$$

Where $\hat{x}_{t-1}$ is the filtered state vector given by

$$\begin{cases} \tilde{x}_t = \hat{x}_t = M_t H_t g_t(\bar{z}_t) \\ M_{t+1} = \Phi_t P_t \Phi_t^{'} + Q_t \\ P_t = M_t - M_t H_t^{'} G_t(\bar{z}_t) H_t M_t \end{cases}$$

Where $\bar{z}_t = z_t - \hat{z}_t$ is the innovations vector, $g_t(\bar{z}_t)$ is the score function of the innovations with components and $G_t(\bar{z}_t)$ is defined as the differential of the score function.

2.4.2 High Frequency Trading: an Application

As an example of High Frequency strategy we will study a spread trading model based on Bollinger Bands.

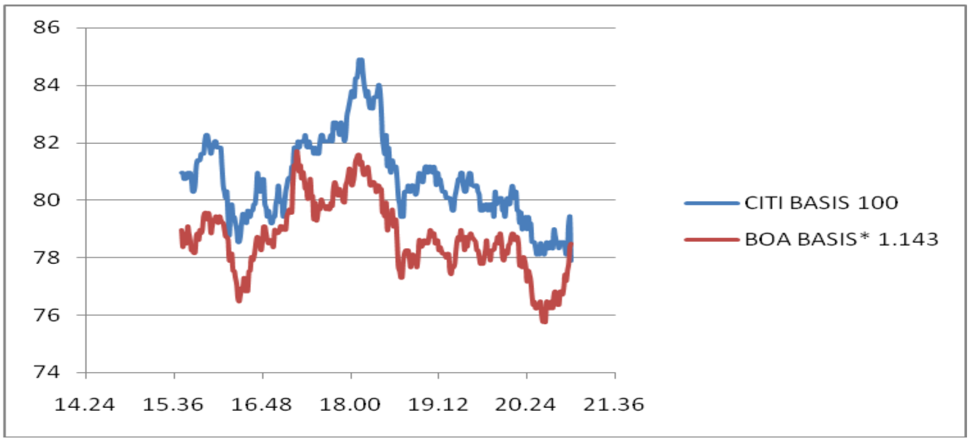
They are a technical analysis tool useful to provide a definition of high prices (the upper limit $UL(t)$) and low prices (the lower limit $DL(t)$), defined as follows

$$DL(t) = MA(t) - k * VOLA(t)$$
$$UL(t) = MA(t) + k * VOLA(t),$$

Where $MA(t)$ is the moving average over the last n observations, $VOLA(t)$ is the volatility over the same time horizon and *K* is the volatility multiplier.

A common entry (exit) level is given by the market price crossing the upper (lower) limit of the bands.

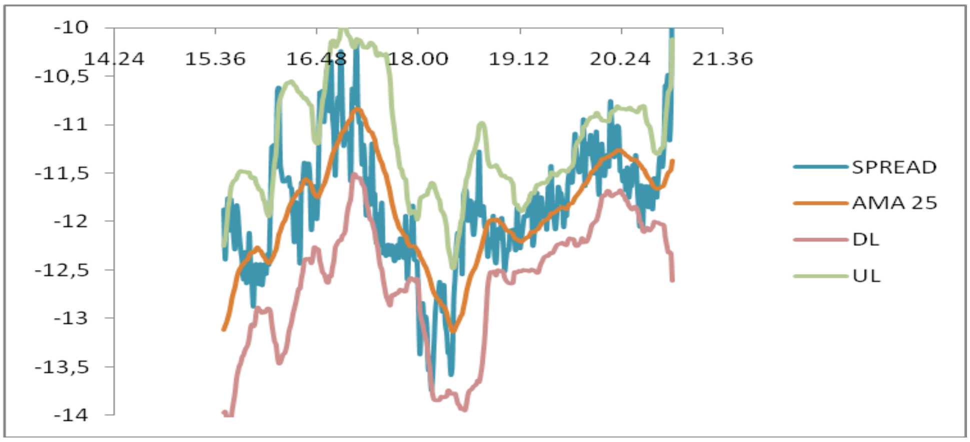
In the example it is studied the spread Citigroup-Bank of America



with a sample length of 3 months and a frequency of 1 minute: The strategy results are summarized below

	Return Mean	Return Volatility	Return-Risk Ratio
High Frequency Strategy	1.68%	0.85%	1.98
HFR Equity Market Neutral	3.03%	2.88%	1.054

The graphical representation of the Bollinger Bands obtained is reported below.



2.5 Behavioral SA

Behavioral Statistical Arbitrage is that branch of SA which specializes in exploiting market inefficiencies related to behavioral phenomena. It represents an application of behavioral finance (and mostly some areas of it, like the so called psychology). Researchers in psychology behavioral finance studies the different kind of deviations from full rationality (among them, representativeness and overconfidence). For a more detailed discussion about behavioral finance please refer to the Chapter 1.

2.5.1 Contrarian Strategy

Contrarian Strategies represent the most immediate application of behavioral finance for SA, as they aim to exploit market inefficiencies, motivated by behavioral theory such as the above mentioned representativeness or overconfidence (overreaction). These strategies can be seen as the opposite of momentum strategies, where usually investors tend to buy securities which performed well, over the most recent time horizons, selling those stocks which had very poor results.

The key steps of the strategy are

1. identification of the criteria to select the securities (stock selection)
2. long/short portfolio construction
3. holding period (and trading levels)

Stock Selection. Stocks can be selected on the back of different criteria. Among those, many practitioners simply use past returns. However, other indicators (many from technical analysis) can be useful, such as: momentum strength, volume patterns, sales growth, earnings growth and cash flow growth. In relation to the choice of the time horizon of the analysis many authors agree about considering a ranking period ranging from 6 months to 1 year.

Long/Short Portfolio Construction. Many authors suggest holding an equally weighted portfolio where the long side is perfectly matched by the short side in order to have a zero cost strategy.

Sudak and Suslova (2003) report three asset allocation techniques:

1. Portfolio Variance Minimization
2. Covariance Minimization
3. Optimization with zero Beta

Portfolio variance aims to minimize the portfolio variance with the portfolio weights w_i, such that $\sum_{i=1}^{N} w_i^L + \sum_{j=1}^{M} w_j^S = 0$, where N and M is the number of stocks in the long and short portfolio, w^L indicates the long portfolio allocation, and w^S indicates short portfolio allocation. Analogously, covariance minimization produces weights that minimize the portfolio covariance between the long and short side. At last, optimization with beta equal to zero aims to build a portfolio which is market neutral. Please note that in the portfolio optimization there is no mention to expected returns as we are considering contrarian strategies (i.e. with poor recent performances).

Holding period and trading levels. The literature in relation to the technicalities of the trading rules is quite poor as trading desks do not easily disclose this type of information. Some authors recommend holding periods from 1 month to 1 year.

2.5.2 Value Portfolio Trading

Value investing is an SA strategy which exploits market inefficiencies aiming to find undervalued securities.

The stock selection process can use several indicators such as Price/ Sales, Price/ Earnings, Price/ Book Value and Dividend Yield.

Moreover, beyond these ratios, we can use also ratios as PEG, or Price/ Earnings growth adjusted, D/ E, or debt- equity ratio, and current assets/ current liabilities.

The Long/Short portfolio construction process works in similar fashion to that one of a contrarian strategy.

2.5.3 Behavioral SA: Applications

As a first example we consider a simple strategy given by buying undervalued S&P 500 bank stocks while selling at the same time other overvalued bank stocks.

To aim this goal, we consider the money center banks below: Bank Of New York, Bank Of America, Citigroup, JP Morgan Chase, Keycorp, P N C Fin, SunTrust Banks, Wells Fargo.

The strategy will select value securities (according to the P/E) with momentum based trading criteria.

The sample period ranges from 2003 till 2009 with a 3 month ranking period and a 12 month holding period.

At last, the trading strategy is compared with the HFR Equity Market Neutral index risk-return profile.

From	Till	Long	Short
01/04/2003	01/04/2004	Keycorp SunTrust Bank of America	Bank of New York Citigroup Wells Fargo
01/04/2005	01/04/2006	Citigroup Keycorp	Bank of America SunTrust
01/04/2006	01/04/2007	Keycorp	Citigroup
01/04/2007	01/04/2008	Bank of America Citigroup JP Morgan	Keycorp SunTrust Wells Fargo
01/04/2008	01/04/2009	Keycorp Wells Fargo	Citigroup SunTrust

With the following results

	Return Mean	Return Volatility	Return-Risk Ratio
Value Strategy	32.59%	35.26%	0.924
HFR Equity Market Neutral	3.03%	2.88%	1.054

As a second example we consider a strategy given by shorting S&P 500 bank stocks with positive momentum while taking an opposite position on others bank stocks.

The trades will be opened when a bank has the one month positive momentum higher than the three month positive momentum.

The sample period ranges from 2003 till 2009 with a 3 month ranking period and a 12 month sample period.

At last, the trading strategy is compared with the HFR Equity Market Neutral index.

From	Till	Long	Short
01/04/2003	01/04/2004	Bank of America Keycorp SunTrust	Citigroup JP Morgan Wells Fargo
01/04/2005	01/04/2006	Bank of New York Bank of America JP Morgan	Wells Fargo Keycorp SunTrust
01/04/2007	01/04/2008	SunTrust	Keycorp
01/04/2008	01/04/2009	JP Morgan Wells Fargo Keycorp	Citigroup SunTrust Bank of America

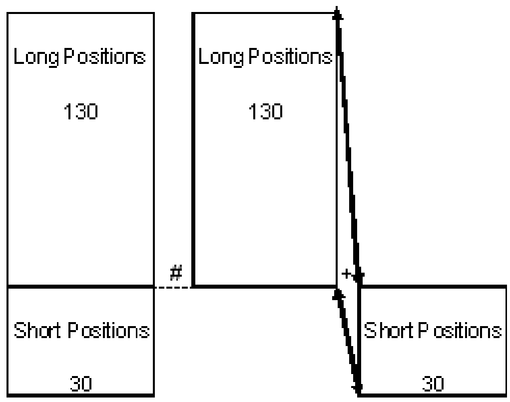
With the following results

	Return Mean	Return Volatility	Return-Risk Ratio
Contrarian Strategy	74.80%	157.73%	0.474
HFR Equity Market Neutral	3.03%	2.88%	1.054

CHAPTER 3- STRATEGY APPLICATION

3.0 Overview

In this chapter we will review one of the most innovative areas in the asset management industry, the so called 130/30 products. These are mutual funds allowing portfolio managers to hold simultaneously both long (up to 130%) and short (up to -30%) positions on different securities. Because of that, 130/30 products have features in common with the hedge funds but with a lower risk profile typical of the retail products.



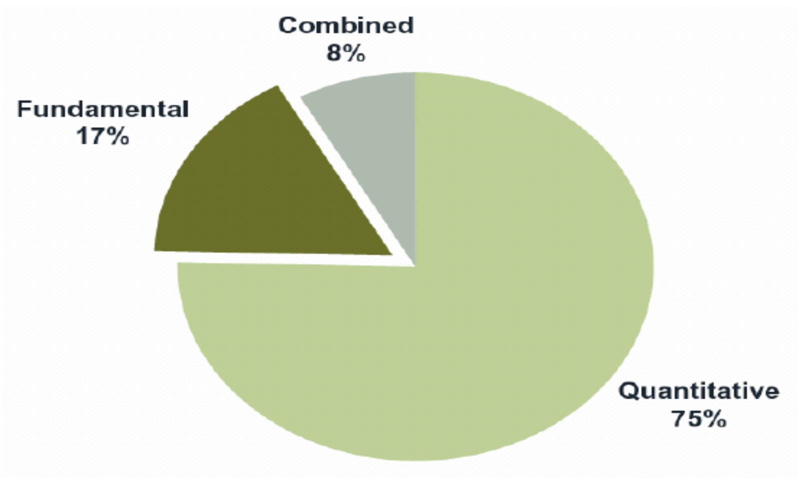
We will provide a simplified application of a SA strategy to a 130/30 product. That will be done not with physical shortening but through a synthetic equity swap, an instrument allowing to take both short and long positions in exchange of periodical variable payments based on the swap notional amount.

3.1 130/30 Products

Enhanced long or 130/30 strategies have grown in popularity in recent years. These strategies differ from traditional long-only products by virtue of the fact that they can hold short equity positions (usually achieved using derivatives rather than physically short-selling of the asset), typically up to 30%. The proceeds from these short positions are then invested in additional long positions so that the long portion of the portfolio has a value close to 130% of the fund's NAV. The long (130%) and the short (-30%) positions are usually combined to retain an overall beta close to 1.

A market overview shows that, at August 2008, the estimated AUM (Assets Under Management) was of $ 120 billion globally, of which 80% in the US, 15% in Europe and 5% in Asia (Source Credit Suisse). The available breakdown of the funds for 2008 highlights how about 75% of the funds is managed through quantitative techniques while only 17% is based on fundamental analysis.

Breakdown of 130/30 funds, 2008



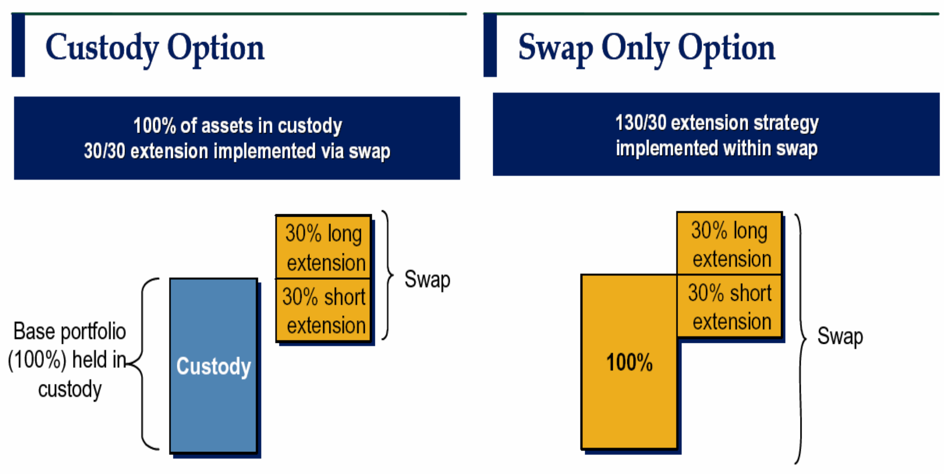
Source: Pensions & Investments; Merrill Lynch

3.1.1 130/30 Products Structure

The most common structure for a 130/30 product is via a swap constructed by a third party (known as a Synthetic Prime Broker). The Synthetic Prime Broker usually combines leverage, financing, stock lending, execution and settlement. Two options are usually offered:

Custody Option. This is where the traditional 100% long-only part of the fund is managed with the assets held by the fund's custodian. The 30/30 extension is then implemented via the swap.

Swap-only Option. In this case, the entire 130/30 strategy is implemented within the swap. The main advantage of this approach is its operational simplicity.

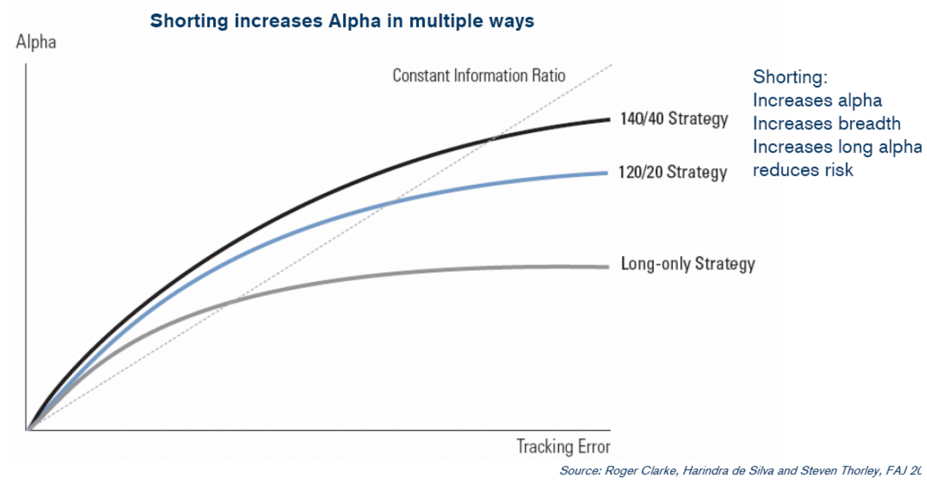


The decision as to the exact financing structure adapted by the fund depends on several criteria including fund type, NAV, projected turnover and ability to trade short positions.

3.1.2 130/30 Products versus Long Only Products

Advantages. Recent studies have shown that the restriction to long-only investment eliminates almost half of the trading opportunities available to a portfolio manager. By relaxing the long-only constraint, 130/30 funds can exploit negative stock views allowing portfolio managers to increase their Information Ratio along two dimensions:

1. the number of active positions (sometimes defined as *breadth*)
2. the size of the manager's bets.



They key feature of 130/30 funds is anyway that they allow short exposure (up to 30%) while still maintaining a beta close to one like the long-only products.

Disadvantages. In comparison to long-only products, the main disadvantages of 130/30 funds are the higher costs and the need of more sophisticated reporting systems.

By implementing a portfolio based on 130/30 strategies, costs are increased in comparison with long only strategies.
Indeed shorting securities generates additional costs.
The two main costs which can be considered are shorting costs and execution (or trading) costs.

3.1.3 130/30 versus Hedge Funds

For many investors, 130/30 products are hybrid structures which shares several similarities with hedge funds, both in terms of investment style and structure.

Here below we show a table reporting some of them (Source Credit Suisse).

It is a hedge fund!	It is a beta-one equity fund!
▪ Shorting	▪ Beta of 1
▪ Leverage	▪ Strict constraints
▪ Extra due diligence required	▪ Falls into equity allocation
▪ Higher risk than long-only	▪ Lower fees than hedge funds
▪ Operational considerations	▪ Benchmarked funds

3.2 Synthetic Equity Swaps

Equity swaps are OTC (Over The Counter) financial instruments. They have been used in alternative investment since 1990s and originally they were introduced to meet hedge funds speculative needs. In more recent times and particularly after the introduction of the UCITS III (Undertaking for Collective Investment in Transferable Securities), the use of synthetic products have become more popular even for retails products and the traditional asset management industry.

Like stock index futures, equity swaps are linear on the underlying assets but, differently from futures, usually they do not have (close) maturity dates.

Differently from a direct investment in the underlying security, equity swaps allow leverage and shorting.

Differently from futures equity swaps show a much greater flexibility as they can be written on customized basket of securities or indices with lower constraints in terms of contract size. Due to the lack of a maturity, they do not incur in rolling costs.

Market participants usually refer to synthetic equity index swaps as to those swaps which synthetically deliver the performance of an equity index (or a basket of securities) in exchange of a floating rate.

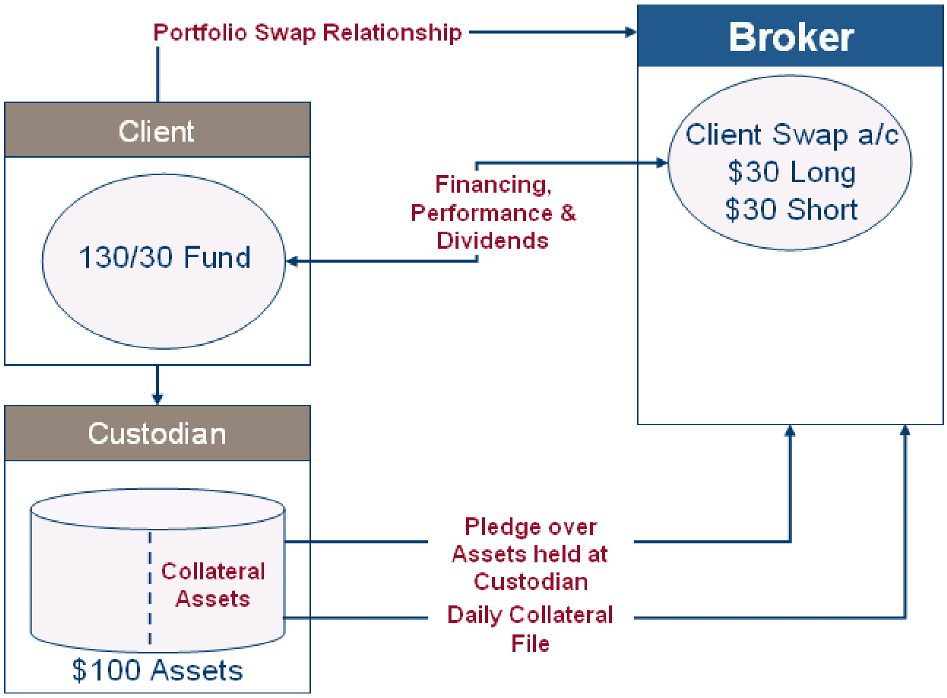
3.2.1 Synthetic Equity Swap: Structure in 130/30

According to the most common structure, the **equity swap buyer** pays a variable amount to receive the performance of a security. The variable amount is usually given by a floating rate (Euribor) increased by a spread depending on the underlying security. The so defined rate will be applied to the notional outstanding for the relevant period. In formulas

$$Payment_{buyer} = (Euribor + Spread) \cdot Notional \cdot \frac{\Delta t}{360} \tag{7}$$

The **equity swap seller** instead will deliver the performance of the underlying securities in exchange of the payment.

Below it is reported a common structure of Equity (Portfolio) swap relationship in a 130/30 context.



3.3 Pairs Trading with Synthetic Index Swap for 130/30 Products

In this example of application we will test the profitability of a cointegration approach pairs trading strategy for a 130/30 product through the use of a synthetic equity swap. In the example we will use a spread of 30 bps for the equation (7). This spread reflects current market values as of February 2009.

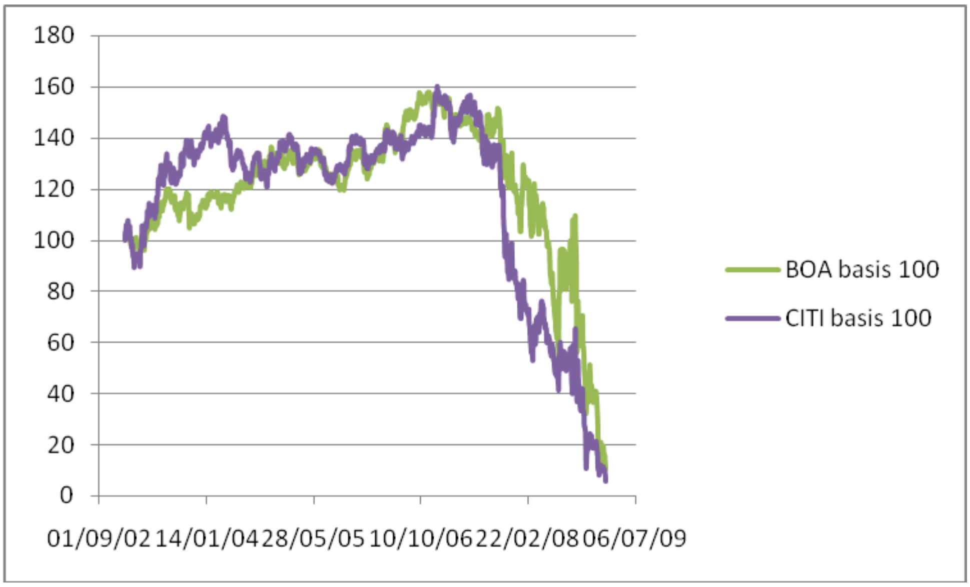
The sample period ranges from 01/01/2003 till 30/11/2008 while the out of sample period from 01/12/2008 till 20/02/2009.

The Pairs Trading strategy considered will be characterized by three components, a beta trading strategy, a cash trading strategy and an alpha strategy. The beta strategy buys and hold at 90% the S&P 500 either investing directly in the underlying equities or other instruments such as ETFs or certificates. The cash strategy invests and hold 10% in 10 year T-Bond.

	Mean	Volatility
Beta Strategy	-8.29%	21.71%
Cash Strategy	0.63%	2.08%

The alpha strategy overlays a -30% /+30% long/short pairs given by Bank of America (BOA) - Citigroup (CITI).

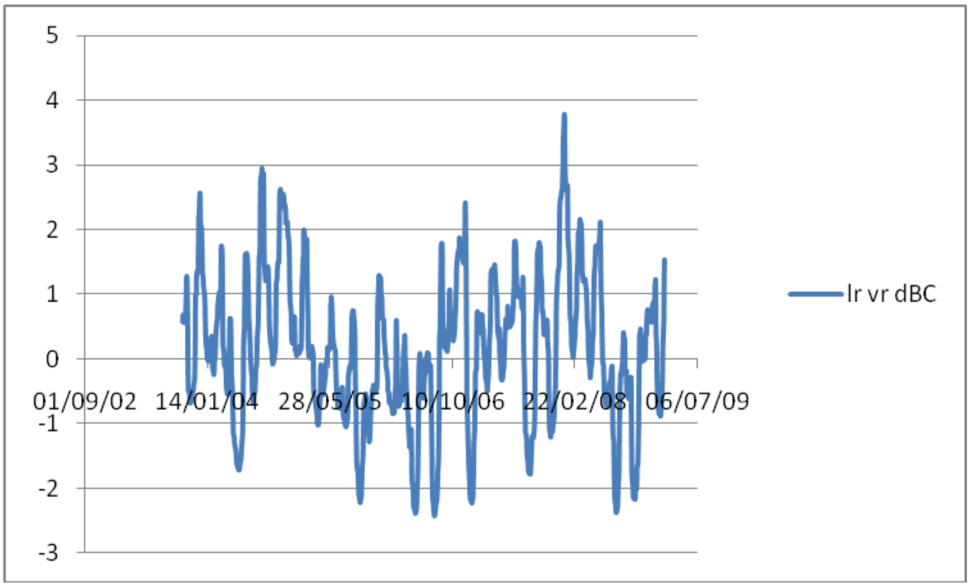
In order to detect the presence of cointegration a first graphical analysis has been carried out, showing similar patterns.



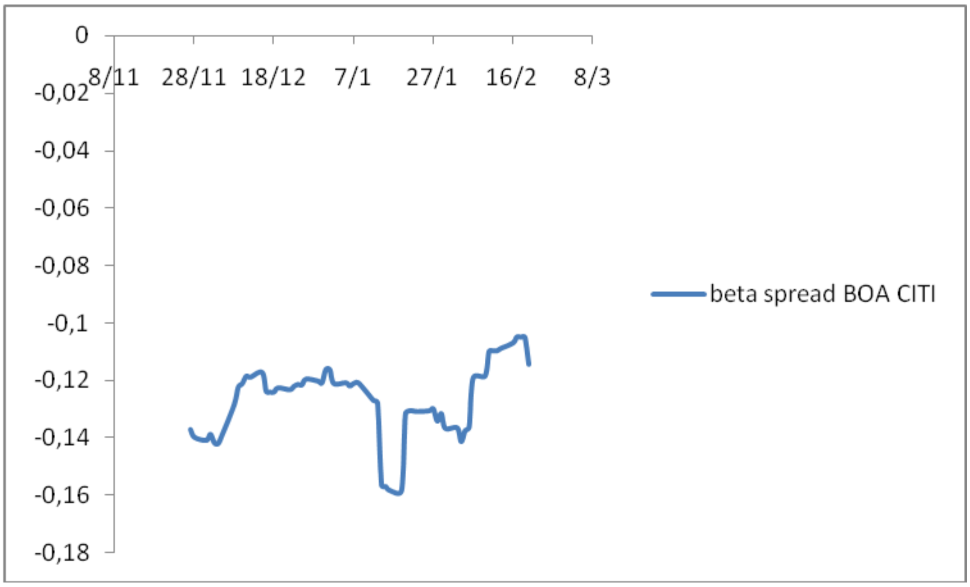
Then evidences of cointegration are found using an Engle-Granger framework. Here we report some results

DF Statistic		ADF statistic	
$\Delta Y_t = AY_t + u_t$	-41.216	$\Delta Y_t = AY_t + B\sum_{i=1}^{n}\Delta Y_{t-i} + u_t$	-29.851
$\Delta Y_t = A + BY_t + u_t$	-41.204	$\Delta Y_t = A + BY_t + C\sum_{i=1}^{n}\Delta Y_{t-i} + u_t$	-29.842
$\Delta Y_t = A + Bt + CY_t + u$	-41.205	$\Delta Y_t = A + Bt + CY_t + D\sum_{i=1}^{n}\Delta Y_{t-i} + u_t$	-29.861

A Variance ratio has been calculated, and it shows the following pattern



At last, an ECM(p,q) analysis is performed. After the definition of the strategies and the detection of cointegration, it has been performed a check that the beta of the pair spread is between -0.2 and 0.2. The graphical depiction is the following



Now all the elements confirm that is possible to implement the strategy which (like in the previous paragraphs) will be opened when the ratio hits a 2 standard deviation band from the mean for two consecutive times on 130 days.

The position will be closed when the ratio hits the mean (in Take Profit) or when the strategy loss reaches 2% (in Stop Loss).

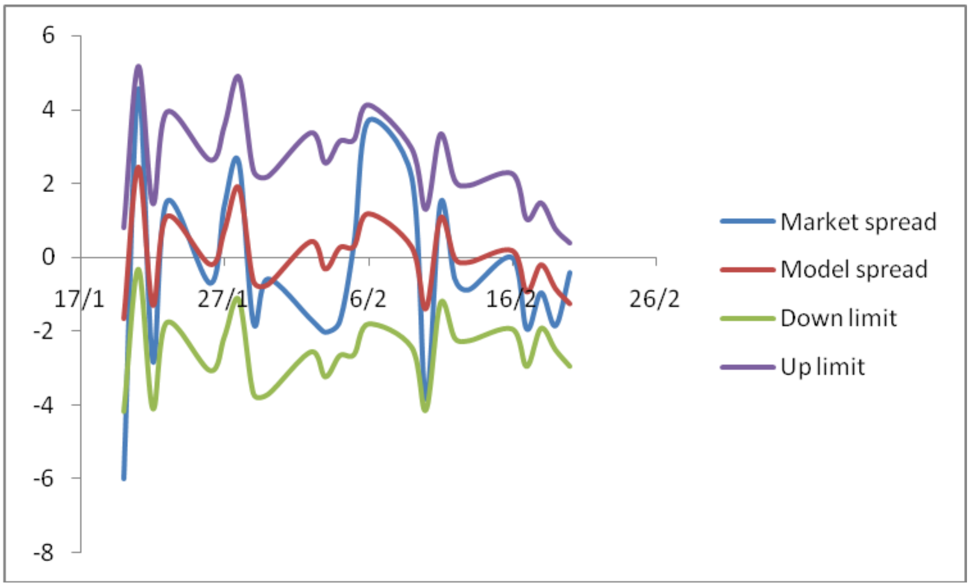
The positions opened are

30/12/2008-31/12/2008 buy signal, 5.843% return
12/01/2009-13/01/2009 sell signal, 11.979% return
15/01/2009-19/01/2009 sell signal, 5.394% return
19/01/2009-21/01/2009 sell signal, 11.645% return
21/01/2009-22/01/2009 buy signal, 0.559% return
06/02/2009-09/02/2009 buy signal, 10.345% return
10/02/2009-11/02/2009 sell signal, 0.583% return

The cumulative results are reported below

	Return Mean	Return Volatility	Return-Risk Ratio
Alpha Strategy	6.62%	4.88%	1.36
Pairs Trading Eq. Swap 130/30	-3.24%	11.85%	-0.27
Beta Strategy (MSCI USA)	-8.06%	22.46%	-0.36

And graphically, between 23/01/2009-20/02/2009



CHAPTER 4- CONCLUSIONS

In this thesis we reviewed some of the today most popular and debated Statistical Arbitrage (SA) strategies. We discussed their application to the highly innovative 130/30 products, where the strategies were applied through the use of synthetic index swaps with up to date real markets spreads.

In the first chapter it was introduced the concept of SA as an attempt at extending the fundamental concept of arbitrage. In order to do it, we reviewed the Efficient Market Hypothesis (EMH) in its different forms and showed as a large number of empirical studies conclude that financial time series appear to contradict them.

We moved then to analyze some SA strategies: Pairs Trading, High Frequency Strategies and Behavioral Strategies.

Pairs Trading is probably the most popular SA strategy mainly used by Hedge Funds. It aims to exploit financial markets inefficiencies by taking positions on a spread between two securities when that spread significantly diverges from its equilibrium levels. Specifically, we studied Pairs Trading according to two different approaches: Stochastic Spread approach and Cointegration Approach. For every technique we provided some examples of applications.

With extremely volatile markets, high frequency data trading is becoming more and more popular. Among the different high frequency tools we reported Technical Analysis and Robust Kalman filtering. Even in this case we provided an application given by the use of Bollinger Bands.

At last we reviewed behavioral strategies on the back of the attention generated by the today increasingly unstable and, possibly, irrational equity markets. Among them we discussed Contrarian strategies (like shorting securities with high momentum) and Value strategies.

In the last chapter we studied real case applications of the strategies to one of the most innovative areas of asset management: 130/30 products. These are a type of mutual fund allowing portfolio managers to hold both long (up to 130%) and short (up to -30%) positions on different securities. For those products SA strategies usually are not applied using physical shortening but through synthetic index swaps (instruments allowing both short and long positions in exchange of periodical variable payments linked to the swap notional). In our simulation we take into account of real market spreads.

The aim of the thesis is to help bridging a gap between research and asset management industry in relation to highly popular techniques. For every strategy we provide the theoretical framework and the strategy mathematical algorithm to finally discuss the technicalities of implementation in real portfolios.

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