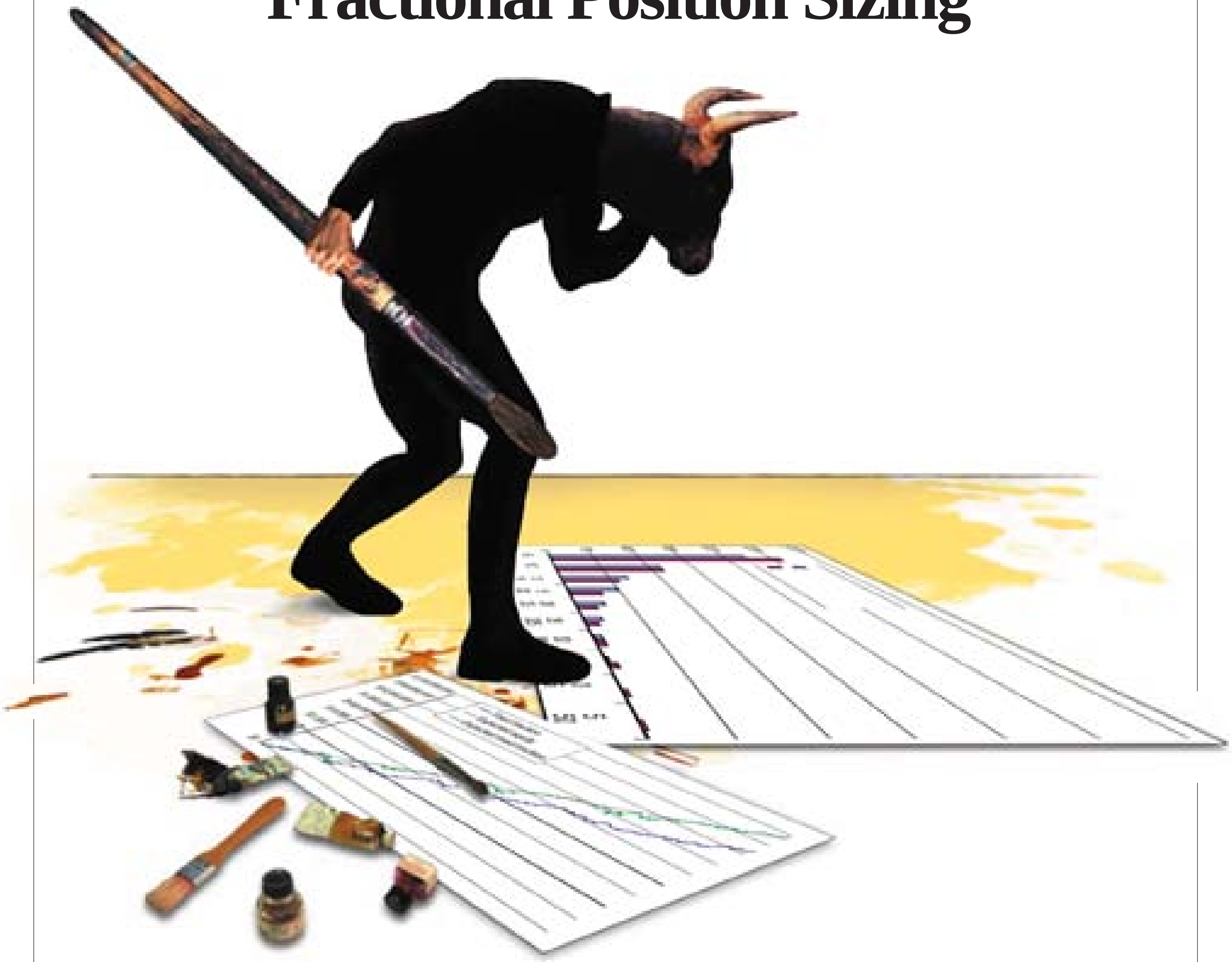
MONEY MANAGEMENT

Is Fixed-Fractional Position Sizing Ill-Fated?

Fixing The Flaws In Fixed-Fractional Position Sizing



JOHN STEWART

Fixed-fractional position sizing is a time-tested method for money management, but in the long run, it will never achieve system expectancy. Here's how you can fix this flaw.

by Christian B. Smart, PhD

Fixed-fractional position sizing is a popular and time-tested method for money management. In the strategy, a fixed percentage of equity is risked per trade. The formula is given here as:

$$\text{Amount risked per trade} = \text{Equity} * f$$

where f is the fixed percentage of equity risked per trade.

Fixed-fractional money management is an intuitive method in which bet size increases when equity increases and bet size decreases when equity decreases. This form of money management is conservative in that it dramatically decreases risk of ruin.

A concept related to money management is *system expectancy*. A system's expectancy is the average, or expected, amount of money an investor expects to make per dollar risked. For example, a trading system with a winning percentage of 40%, whose average win is equal to twice the average loss, has an expectancy approximately equal to 0.40 * 2 – 0.60 = 0.80 – 0.60 = 0.20.

On average, the system returns 20 cents for every dollar risked. If an investor uses fixed-fractional position sizing and risks 2% of equity per trade, then the average expected return per trade is (2%) * 0.20 = 0.40% of equity. The expected equity for an investor with \$100,000 of initial risk capital is \$100,400 after the first trade, \$100,801 (=\$100,400 * 1.004) after the second, and $\$100{,}000 * (1.004)^N$ after the N^{th} trade.

With fixed-fractional position sizing, the system does not achieve this expectancy in the long run, but an amount less than the system expectancy. Risking 2% per trade in a system where all losses are the same size, all wins are the same size, and wins are twice as large as losses, equity either increases by 4% (0.02 * 2) or decreases by 2% (0.02 * 1) on each trade.

After three trades — a win, a loss, and a win — the account equity is increased by (1.04) * (0.98) * (1.04) = 1.059, or approximately 6%. After N trades with M wins and $N - M$ losses, the total return is $(1.04)^{M}(0.98)^{N-M}$. In the long run, M will be 40% of N, so for sufficiently large N, the return will approximately be $(1.04)^{0.4N}(0.98)^{0.6N}$ times the original equity.

With a progressive betting system like fixed-fractional sizing in which returns are reinvested, the total return is the product of a series of numbers. The average of a product of a series of numbers is the geometric mean, which is simply the N^{th} root of the product of N values.

For the series $(1.04)^{0.4N}(0.98)^{0.6N}$, the geometric mean is:

$$\sqrt[N]{1.04^{0.4N}0.98^{0.6N}}$$

which reduces to $1.04^{0.4}0.98^{0.6} = 1.003573$, or a return of 0.3573% per trade. This is less than the system expectancy of 0.40% with 2% risked per trade. While this may not seem like a large difference, it makes a noticeable impact after only a few dozen trades. Figure 1 contains the fixed-fractional expected return vs. the expected equity after 1, 10, 100, and 1,000 trades. The system is losing over six percentage points after only 100 trades with this form of position sizing because of this flaw.

The underperformance of fixed fractional has a basis in mathematics. The system expectancy is the system's arithmetic mean. The average amount made per trade with fixed-fractional position sizing is the system's geometric mean. A well-known inequality in mathematics states that the geometric mean is always less than or equal to its arithmetic mean; so in the long run, fixed-fractional position sizing will never achieve system expectancy but will underperform.

How can this flaw be fixed? The solution is to bet a fixed percentage of *expected* equity instead of actual equity, a strategy I call *expected fixed-fractional*. This allows the system to be exploited to achieve the full expectancy rather than underachieving, as with fixed-fractional. The formula for the amount to risk on the N^{th} trade in a series with expected fixed-fractional position sizing is given in the equation here:

$$\text{Amount risked per trade} = \text{Expected equity} * \text{fixed-fraction}$$

where $\text{expected equity} = \text{Initial equity} * (1 + \text{system expectancy} * \text{Fixed-fraction})^{N}$

Number of trades	System expectation	Fixed-fractional expectation
1	0.4%	0.3573%
10	4.073%	3.631%
100	49.063%	42.813%
1,000	5,316%	3,429%

FIGURE 1: COMPARING FIXED-FRACTIONAL EXPECTATION WITH SYSTEM EXPECTANCY

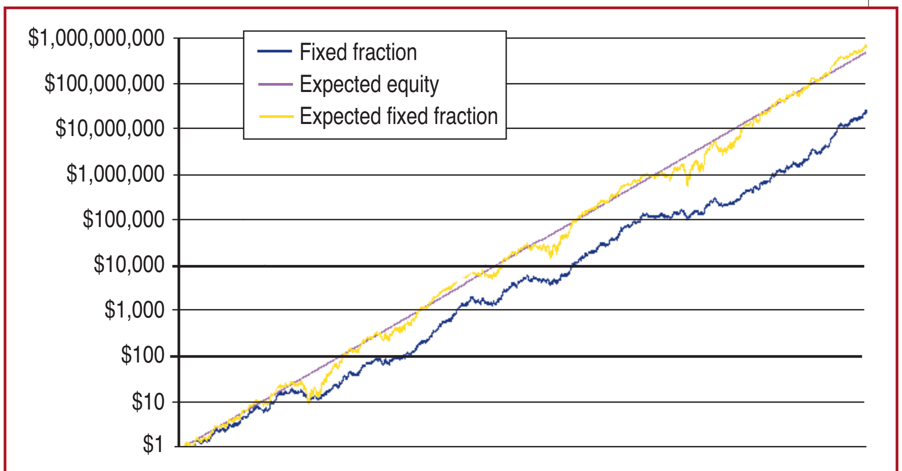


FIGURE 2: COMPARISON OF EXPECTED FIXED-FRACTIONAL AND FIXED-FRACTIONAL FOR 5,000 TRADES

Figure 2 contains a comparison of fixed-fractional and expected fixed-fractional position sizing, based on a 5,000-trade Monte Carlo simulation of the system. In Figure 1, note how expected fixed-fractional position sizing varies about the expected equity curve, instead of below it as with fixed-fractional.

After 5,000 simulated trades, fixed-fractional position sizing turns $1 into $66 million, while expected fixed-fractional position sizing turns $1 into $617 million, a factor of 10 difference and higher than the expected equity of $466 million.

AT WHAT PRICE?

The one disadvantage of expected fixed-fractional is that this increase in returns comes at a price: maximum drawdowns that are worse than fixed-fractional in most cases. For example, for the 5,000-trade Monte Carlo simulation, the maximum drawdown for fixed-fractional position sizing is approximately 40%, while the maximum drawdown for expected fixed-fractional position sizing is approximately 50%. This occurs because fixed-fractional is more conservative.

During a drawdown, if actual equity is less than expected equity, fixed-fractional position sizing risks less per trade than expected fixed-fractional position sizing, which bases the amount risked on expected equity.

Figure 3 contains a histogram of the drawdowns experienced by the two position-sizing methods for the 5,000-trade

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Expected fixed-fractional position sizing clearly provides better returns in the long run.

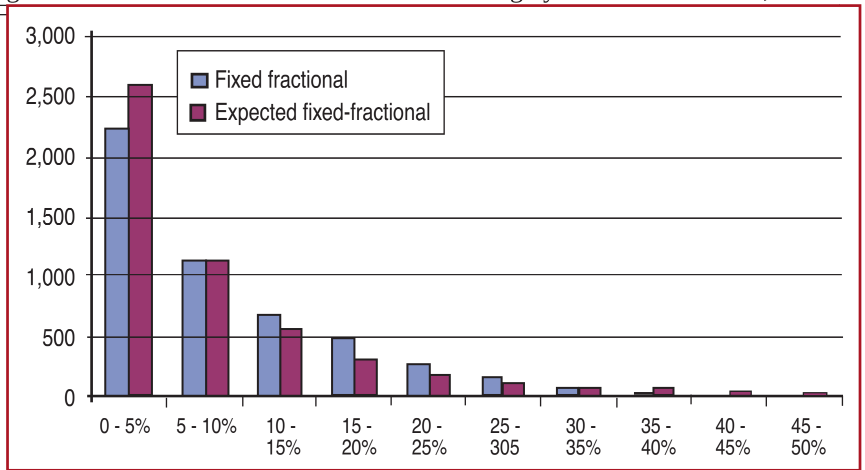

FIGURE 3: HISTOGRAM OF DRAWDOWNS WITH EXPECTED FIXED-FRACTIONAL AND FIXED-FRACTIONAL FOR 5,000 TRADES

simulation. In the histogram, expected fixed-fractional position sizing has more drawdowns in the zero to 5% range, including 962 new equity highs versus only 641 for fixed-fractional. But fixed-fractional's worst drawdown is less than 40%, while expected fixed-fractional's has 44 individual instances with drawdowns between 40% and 50%.

These higher drawdowns highlight a potential shortcoming of expected fixed-fractional trading. Over time, actual equity and expected equity can differ by bigger amounts, leading to increasingly larger amounts of equity being risked. Care must be taken when applying this methodology to not risk too much equity.

Expected fixed-fractional betting also works well with other systems. Figure 4 contains a comparison of fixed-fractional position-sizing versus expected fixed-fractional for three systems for a 5,000-trial Monte Carlo simulation. For simplicity's sake, the beginning equity is set equal to $1. All three systems have an expectancy equal to 0.2 and the winning percentage varies from 40% to 60%.

Average win/average loss	2	1.4	1
Win %	40%	50%	60%
Expectancy	0.2	0.2	0.2
Fixed-fraction	2%	2%	2%
Geometric mean	1.003573	1.003713	1.003808
Arithmetic mean	1.004	1.004	1.004
Expected equity	$466,191,172	$466,191,172	$466,191,172
Fixed-fraction ending equity	$66,424,373	$111,680,122	$186,226,194
Exp. fixed-fraction ending equity	$617,693,297	$316,958,231	$449,585,531
Max drawdown fixed-fractional	-40.36%	-33.19%	-25.00%
Max drawdown Exp. Fixed-Fractional	-50.57%	-34.17%	-24.01%

FIGURE 4: COMPARISON OF EXPECTED FIXED-FRACTIONAL WITH FIXED-FRACTIONAL FOR THREE DIFFERENT SYSTEMS

For all three systems, after 5,000 simulated trades, the equity with expected fixed-fractional position sizing was substantially better than fixed-fractional position sizing by a factor of 10 to 2. For the systems with winning percentages of 50% and 60%, the maximum drawdowns are approximately the same for both fixed-fractional and expected fixed-fractional position sizing, so for these cases, expected fixed-fractional position sizing also provided superior risk-adjusted returns.

Expected fixed-fractional position sizing clearly provides better returns in the long run than comparable fixed-fractional position sizing and can be a powerful way for an investor to boost returns.

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