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A WAVE THEORY FOR NON-IMAGING CONCENTRATORS

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ABSTRACT

An expression is derived for propagation of the radiance function for light in any state of coherence through a concentrator that can be represented by a linear, stationary optical system. For light from a quasi-homogeneous source, this expression can be somewhat simplified by an approximation. It is shown that the radiance function is invariant for a large class of optical systems. It is also shown that fundamental limitations for the concentration of light follow from the uncertainty principle and the second law of thermodynamics, which apply quite generally. These relations show why quasi-homogeneous light (such as light from thermal sources) can not be concentrated as well as light from some other sources (such as light from a laser).

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ILLUSTRATIONS

1. INTRODUCTION

Non-imaging concentrators are almost always studied using geometrical optics,[1] because no wave theory techniques have been developed that are adequate for this purpose. Recently, some progress has been made toward developing a wave theory that might be useful.[2,3,4] A primary difficulty is the proper definition of the radiance function within a wave theory. Walther[5,6] has suggested two different definitions, which have received some study.[7,8] In this report, some preliminary observations are made on the way such a theory might be used to analyze non-imaging concentrators.

The concentration of light of arbitrary coherence is considered in section 2. An equation is derived that describes the propagation of the radiance function for such a light field through any linear, stationary optical system. Some limitations for the concentration of light, which are usually obtained from geometrical optics, are derived using this wave theory. In section 3, the special case of light from a quasi-homogeneous source is considered, and it is found that the propagation expression for the radiance function is simplified. For a class of linear, stationary optical systems (including free space propagation in a short-wavelength limit), the radiance function is invariant upon propagation of the light through the system.

2. RADIATION IN ANY STATE OF COHERENCE

We will begin by considering a light field, of arbitrary coherence, propagating away from the $z = 0$ plane to the right, as shown in Figure 2.1. For convenience, the energy transported by each monochromatic component of this field is described using the complex form of the radiance function over the $z = 0$ plane, as defined by Walther (W),[6]

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[1]W. T. Welford and R. Winston, *High Collection Nonimaging Optics*, Academic Press, New York (1989).

[2]W. Welford and R. Winston, "Generalized Radiance and Practical Radiometry," *J. Opt. Soc. Am. A: Opt. Image Sci.* 4, 545–547 (1987).

[3]R. Winston and X. Ning, "Generalized Radiance of Uniform Lambertian Sources," *J. Opt. Soc. Am. A: Opt. Image Sci.* 4, 516–519 (1988).

[4]R. Winston, "Analogy Between Fresnel Diffraction and Generalized Radiance," *J. Opt. Soc. Am. A: Opt. Image Sci.* 6, 145–146 (1989).

[5]A. Walther, "Radiometry and Coherence," *J. Opt. Soc. Am.* **58**, 1256–1259 (1968).

[6]A. Walther, "Radiometry and Coherence," *J. Opt. Soc. Am.* **63**, 1622–1623 (1973).

[7]J. T. Foley and E. Wolf, "Radiometry as a Short Wavelength Limit of Statistical Wave Theory with Globally Incoherent Sources," *Opt. Commun.* **55**, 236–241 (1985).

[8]J. T. Foley and E. Wolf, "Radiance Functions of Partially Coherent Fields," *J. Mod. Opt.* **38**, 2053–2068 (1991).

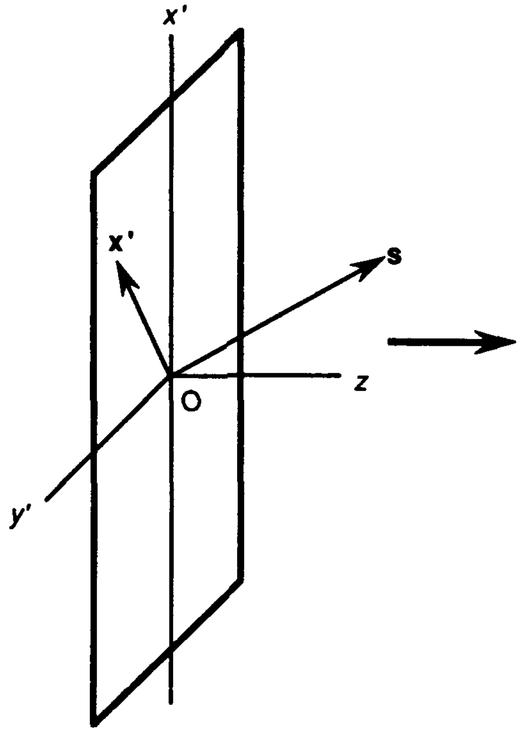

Figure 2.1 An illustration of the coordinate system.

$$B_W^{(0)}(\mathbf{x}_2', \mathbf{s}) = \frac{\cos\theta}{\lambda^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_\omega^{(0)}(\mathbf{x}_1', \mathbf{x}_2') \exp\left[-ik\mathbf{s}\cdot(\mathbf{x}_1' - \mathbf{x}_2')\right] d^2\mathbf{x}_1' , \qquad (2.1)$$

where $W_\omega^{(0)}(\mathbf{x}_1', \mathbf{x}_2')$ is the cross-spectral density function[9] for a typical monochromatic component of the field at frequency ω between two points on the $z = 0$ plane defined by the radius vectors $\mathbf{x}_1'$ and $\mathbf{x}_2'$ from the origin, $\mathbf{s}$ is a unit vector from the origin, θ is the angle that $\mathbf{s}$ makes with the $+z$ axis, and $k = \omega/c$. The radiance function, as usually defined in radiometry, is the energy radiated from a unit area about a point $\mathbf{x}_2'$ in a plane into a unit of solid angle about some direction $\mathbf{s}$ from the origin. In a wave theory it is not possible[10] to define a function that has all of the properties of the geometrical optics radiance function. There is, however, more than one definable function that approximates the geometrical optics radiance function in some useful respects. The definition for the radiance function in Equation 2.1 differs from that defined earlier by Walther[5] and used extensively by Marchand and Wolf[11] in their development of the statistical theory of radiometry and radiative transfer. No reason has yet been discovered to prefer one definition over the other, except its usefulness in a particular theory.

[9]W. H. Carter, "Coherence Theory," in *The Optical Society of America Handbook of Optics*, M. Bass (ed.), McGraw-Hill, New York (in press).
[10]A. T. Friberg, "On the Existence of a Radiance Function for Finite Planar Sources of Arbitrary States of Coherence," *J. Opt. Soc. Am.* **69**, 192–198 (1979).
[11]E. W. Marchand and E. Wolf, "Radiometry with Sources of any State of Coherence," *J. Opt. Soc. Am.* **64**, 1219–1226 (1974).

In Equation 2.1 the radiance is defined as a function of the cross-spectral density function over the $z = 0$ plane, which can be defined by[9]

$$W_\omega^{(0)}(\mathbf{x}_1', \mathbf{x}_2') = \left\langle \psi^{(0)}(\mathbf{x}_1', \omega)\left[\psi^{(0)}(\mathbf{x}_2', \omega)\right]^* \right\rangle \tag{2.2}$$

where $\psi^{(0)}(\mathbf{x}', \omega)$ is a random variable describing the complex amplitude of a typical monochromatic component of the field with radial frequency ω at a point given by the radius vector $\mathbf{x}'$ in the $z = 0$ plane. The angle brackets denote an ensemble average.

If we assume that the sources of the field are to the left of the $z = 0$ plane, as shown in Figure 2.1, then it follows that the amplitude of the field anywhere to the right of this plane can be represented using an angular spectrum of plane waves in the form

$$\psi(\mathbf{x}, \omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_\omega^{(0)}(p, q) \exp\left[ik(px + qy + mz)\right] dp\, dq\,, \tag{2.3}$$

$$m = \sqrt{1 - p^2 - q^2}\,, \qquad \text{if } p^2 + q^2 \le 1\,,$$

$$m = \sqrt{p^2 + q^2 - 1}\,, \qquad \text{if } p^2 + q^2 > 1\,,$$

where the complex amplitude of the plane wave propagating in the direction given by the unit vector $\mathbf{p} = (p, q, m)$ is the Fourier transform of the field amplitude over the $z = 0$ plane; that is,

$$A_\omega^{(0)}(p, q) = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi^{(0)}(\mathbf{x}', \omega)\, \exp\left[-ik(px' + qy')\right] dx'\, dy'\,, \tag{2.4}$$

which has the inverse

$$\psi^{(0)}(\mathbf{x}', \omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_\omega^{(0)}(p, q)\, \exp\left[ik(px' + qy')\right] dp\, dq\,. \tag{2.5}$$

Substituting from Equation 2.5 into Equation 2.1, we have a very simple expression for the radiance function of this field,

$$B_W^{(0)}(\mathbf{x}', \mathbf{s}) = \left\langle \left[\psi^{(0)}(\mathbf{x}', \omega)\right]^* A_\omega^{(0)}(s_x, s_y) \right\rangle \exp(ik\mathbf{s}\cdot\mathbf{x}') \cos\theta\,, \tag{2.6}$$

where the components of the unit vector are $\mathbf{s} = (s_x, s_y, s_z)$. From Equation 2.6, we see that Walther's radiance function for a light field of arbitrary coherence is simply the ensemble average of the product of the complex conjugate of the field amplitude over the $z = 0$ plane, the angular spectrum of plane waves for the field, a Fourier kernel between the conjugate variables $\mathbf{s}$ and $\mathbf{x}'$, and the cosine of θ, the angle that $\mathbf{s}$ makes with the $+z$ axis. It is clear from Equations 2.4, 2.5, and 2.6 that the variables $\mathbf{s}$ and $\mathbf{x}'$, on which Walther's radiance function depends, are Fourier conjugate variables.

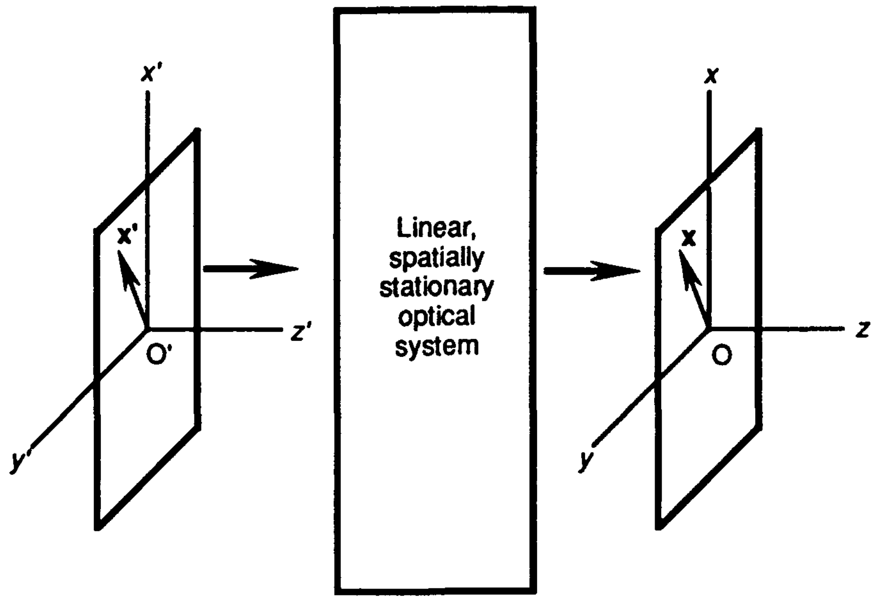

Figure 2.2 Propagation of the radiance function through a linear, spatially stationary optical system.

To treat non-imaging concentrators, we must consider the propagation of the radiance function itself through some arbitrary optical system from some input plane $z' = 0$, (shown to the left in Figure 2.2) to an output plane $z = 0$ (shown to the right in Figure 2.2).

The radiance function on the input plane is given by Equation 2.6, using primed coordinates:

$$B_{\mathrm{W}}^{(z'=0)}(\mathbf{x}',\mathbf{s}') = \left\langle \left[\psi^{(z'=0)}(\mathbf{x}',\omega)\right]^{*} A_{\omega}^{(z'=0)}\left(s'_x, s'_y\right)\right\rangle \exp(ik\mathbf{s}'\cdot\mathbf{x}')\cos\theta' \tag{2.7}$$

The radiance function on the output plane is given by the same equation with unprimed coordinates:

$$B_{\mathrm{W}}^{(z=0)}(\mathbf{x},\mathbf{s}) = \left\langle \left[\psi^{(z=0)}(\mathbf{x},\omega)\right]^{*} A_{\omega}^{(z=0)}\left(s_x, s_y\right)\right\rangle \exp(ik\mathbf{s}\cdot\mathbf{x})\cos\theta\,. \tag{2.8}$$

If the optic:l system is linear and stationary, then the field amplitude in the output plane is

$$\psi^{(z=0)}(\mathbf{x},\omega) = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi^{(z'=0)}(\mathbf{x}',\omega)\,h(\mathbf{x}-\mathbf{x}')\,dx'\,dy' \tag{2.9}$$

as a function of the field in the input plane and the point-spread function $h(\mathbf{x})$ for the optical system. By taking the Fourier transform of Equation 2.9, using the convolution theorem,[12] and then using Equation 2.4 twice to represent the Fourier transforms of the fields over the $z = 0$ and $z' = 0$ planes as angular spectra, we get

$$A_\omega^{(z=0)}(p,q) = A_\omega^{(z'=0)}(p,q)\tilde{h}(p,q), \tag{2.10}$$

an expression for the angular spectrum of plane waves for the field in the output space as a function of the angular spectrum in the input space and the transfer function for the optical system. The tilde indicates a two-dimensional Fourier transform, so the transfer function is given by

$$\tilde{h}(p,q) = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\mathbf{x})\exp[-ik(px+qy)]\,dx\,dy, \tag{2.11}$$

the Fourier transform of the point-spread function.

Substituting from Equations 2.9 and 2.10 into Equation 2.8 and using Equation 2.7, we get

$$B_W^{(z=0)}(\mathbf{x},\mathbf{s}) = \frac{1}{\lambda^2}\exp(ik\mathbf{s}\cdot\mathbf{x})\left\{\left[B_w^{(z'=0)}(\mathbf{x},\mathbf{s})\exp(-ik\mathbf{s}\cdot\mathbf{x})\right] * h^*(\mathbf{x})\right\}\tilde{h}(s_x,s_y) \tag{2.12}$$

as an expression for the propagation of the radiance function through the optical system from the input plane to the output plane. Thus, the radiance function transforms by convolution on its spatial coordinate with the point-spread function for this system, and also by multiplication with the transfer function.

Linear systems are especially easy to treat. Depending on the form of the point-spread function, this system may or may not be a non-imaging concentrator. If the point-spread function reasonably approximates a Dirac delta function, then the system images the field over the $z' = 0$ plane on the $z = 0$ plane. If the system represents free space propagation from the $z' = 0$ plane to a $z = 0$ plane, which is parallel to it and some fixed distance away, then Equation 2.9 is simply the Rayleigh diffraction integral of the first kind[13] and Equation 2.12 holds with $h(x)$, given by the well-known Rayleigh kernel from this integral. A large class of non-imaging concentrators are linear and space invariant, and therefore can be treated using this theory to transform the radiance function. However, some non-imaging concentrators transform the radiance function in a much more complicated way. In the most general case, we can not give a specific equation equivalent to Equation 2.12 for the transformation of the radiance function; however, we can identify some restrictions on the transformed radiance function for even the most general concentrator.

Since the two variables $\mathbf{s}$ and $\mathbf{x}$ in Equation 2.8 (and $\mathbf{s}'$ and $\mathbf{x}'$ in Eq. 2.7) are Fourier conjugate variables, the radiance function can not depend on them independently. If we define the standard deviation for each Cartesian component of these variables by equations such as

[12] J. Goodman, *Introduction to Fourier Optics*, McGraw-Hill, New York (1968).

[13] W. H. Carter, "Three Different Kinds of Fraunhofer Approximations: I. Propagation of the Field Amplitude," *Radio Sci.* **23**, 1085–1093 (1988).

$$(\Delta x)^2 = \frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} x^2 \psi^{(0)}(\mathbf{x},\omega)dx\,dy}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \psi^{(0)}(\mathbf{x},\omega)dx\,dy} - \left[\frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} x \psi^{(0)}(\mathbf{x},\omega)dx\,dy}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \psi^{(0)}(\mathbf{x},\omega)dx\,dy}\right]^2,$$

$$(\Delta p)^2 = \frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} p^2 A_\omega^{(0)}(p,q)dp\,dq}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} A_\omega^{(0)}(p,q)dp\,dq} - \left[\frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} p A_\omega^{(0)}(p,q)dp\,dq}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} A_\omega^{(0)}(p,q)dp\,dq}\right]^2, \tag{2.13}$$

then the radiance function is constrained by the well-known uncertainty law for Fourier conjugate variables; that is,

$$\begin{aligned} v_x(z) &= \Delta x \Delta p \geq \frac{\lambda}{2\pi}, \\ v_y(z) &= \Delta y \Delta q \geq \frac{\lambda}{2\pi}, \end{aligned} \tag{2.14}$$

where the phase space volume function

$$V = v_x(z)v_y(z) = \Delta x \Delta y \Delta p \Delta q \geq (\lambda/2\pi)^2 \tag{2.15}$$

has at least some of the properties of the etendue, which is the product of the area of a light beam, normal to its direction of propagation, and the solid angle that the beam subtends with the beam axis, as defined by geometrical optics theories.[1] Since the maximum value for Δp and Δq is unity (p is a unit vector), we see immediately from Equation 2.14 that the smallest possible spot a concentrator can achieve is given by

$$S_c \stackrel{\Delta}{=} \mathrm{Min}(\Delta x\, \Delta y) = \left(\frac{\lambda}{2\pi}\right)^2. \tag{2.16}$$

Equation 2.6 shows that each monochromatic component of the light can be concentrated into a spot with a radius that is actually somewhat smaller than a single wavelength. Such a small spot, however, is not always obtainable, as shown in the following discussion.

Consider the light concentration properties of a general optical system. All we know about this system is that the phase space volume function for the radiance function, as it propagates through the system, can not be decreased. This is always true of any optical system. This law is usually derived for the etendue in geometrical optics theory, but in our wave theory it follows from the second law of thermodynamics. The phase space volume represents an uncertainty associated with the light field, so the uncertainty associated with the output light field can not be smaller than that associated with the input light field:

$$\frac{V^{(\text{in})}}{V^{(\text{out})}} = \frac{\Delta x' \Delta y' \Delta p' \Delta q'}{\Delta x \Delta y \Delta p \Delta q} \leq 1 . \tag{2.17}$$

This law places a fundamental restriction on what any concentrator can accomplish with a given light field.

Let the input light field form a spot centered on the origin over the $z' = 0$ plane, with radii along the x and y axes defined by the standard deviations $\Delta x'$ and $\Delta y'$, respectively. Assume that this spot of light radiates into the $z' > 0$ half-space within numerical apertures projected onto the xz plane and yz plane and defined by the standard deviations $\Delta p'$ and $\Delta q'$. Since p' and q' are the x and y components of the unit vector $\mathbf{p}'$, normal to the plane waves that make up the radiated field, it follows that if $\Delta \mathbf{p}' = \Delta \mathbf{q}'$, then the cone of light is symmetric about the $+z'$ axis and makes the angle θ' with the axis, such that

$$\sin \theta' \triangleq \Delta p' = \Delta q' . \tag{2.18}$$

In general, we will assume that $\Delta p' \neq \Delta q'$, so that the light cone is not symmetrical about the z' axis.

Let the general optical system act as a light concentrator, forming a spot of light centered on the origin in the output plane, with radii defined by the standard deviations Δx and Δy, and radiating into the $z > 0$ half-space within numerical apertures defined by the standard deviations Δp and Δq. We define the concentration of the light in the usual way as the area of the light spot before concentration, divided by the area of the light spot after concentration:[1]

$$C \triangleq \frac{\Delta x' \Delta y'}{\Delta x \Delta y} . \tag{2.19}$$

Substituting for the standard deviations defining the spot size in the input and output spaces from Equation 2.15 into Equation 2.19, we get

$$C \triangleq \frac{V^{(\text{in})}}{V^{(\text{out})}} \frac{\Delta p \Delta q}{\Delta p' \Delta q'} . \tag{2.20}$$

Thus, upon substitution from Equation 2.17 into Equation 2.20, we have

$$C \leq \frac{\Delta p \Delta q}{\Delta p' \Delta q'} , \tag{2.21}$$

a very general constraint on the ability of any optical system to concentrate light, which is mandated by the second law of thermodynamics. Equation 2.21 states that the concentration of the light field can not exceed the increase in the product of the numerical apertures for the projection of the light cone onto the xz planes and yz planes before and after concentration.

For the special case of an input light spot that is symmetric about the z' axis, so that $\Delta p' = \Delta q' = \sin \theta'$, and a symmetrical optical system, so that $\Delta p = \Delta q = \sin \theta$, we find from Equation 2.21 that the concentration is limited by

$$C \le \left(\frac{\sin\theta'}{\sin\theta}\right)^2 , \tag{2.22}$$

where θ and θ' are the angles that the light cones make with the z axis in the output and input spaces, respectively.

The maximum concentration is obtained over the $z = 0$ output plane if the light cone converging onto this plane from the aperture of the concentrator makes a half-angle θ that approaches 90°. Thus, the absolute limit on the concentration that can be achieved by any light concentrator is given by

$$C \le \frac{1}{\sin^2\theta'} . \tag{2.23}$$

This limitation is identical to a very well known limit on the concentration that is usually derived by use of geometrical optics.[1] The present derivation shows that this limit follows from the second law of thermodynamics and is therefore fundamental to any optical system. Equation 2.16 gives the smallest possible spot of light that can ever be formed with any light field. Once we are given a specific light field, however, with some fixed boundary condition over the $z' = 0$ plane for the field amplitude, Equation 2.23 gives the maximum amount by which the given spot can be reduced without violating the second law of thermodynamics (and thereby decreasing the entropy of this particular field).

3. RADIATION FROM QUASI-HOMOGENEOUS SOURCES

Next, we consider the concentration of radiation from quasi-homogeneous sources rather than more general kinds of radiation. Thermal sources and many other naturally occurring (nonlaser) sources are quasi-homogeneous. For radiation from such sources it is more convenient to begin with Walther's earlier definition for the radiance function, which has been used extensively by Marchand and Wolf (MW)[11] in their theory of radiometry. This definition has been shown to be fully equivalent to the definition used in Equation 2.1 for quasi-homogeneous light fields.[14] For a field from a quasi-homogeneous source over the $z = 0$ plane, we define the complex radiance function, as shown in Figure 1, by

$$B_{\mathrm{MW}}^{(0)}(\mathbf{x}'_+, \mathbf{s}) = \frac{\cos\theta}{\lambda^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_\omega^{(0)}\left(\mathbf{x}'_+ + \frac{\mathbf{x}'_-}{2}, \mathbf{x}'_+ - \frac{\mathbf{x}'_-}{2}\right) \exp(-ik\mathbf{s}\cdot\mathbf{x}'_-) d^2\mathbf{x}'_- . \tag{3.1}$$

For a quasi-homogeneous, secondary source over the $z = 0$ plane, the cross-spectral density function over the $z = 0$ plane is given by the product[14]

[14]W. H. Carter and E. Wolf, "Coherence and Radiometry with Quasihomogeneous Planar Sources," *J. Opt. Soc. Am.* **67**, 785–796 (1977).

$$W_{\omega}^{(0)}(\mathbf{x}_1',\mathbf{x}_2') = I^{(0)}(\mathbf{x}_+')\,\mu^{(0)}(\mathbf{x}_-'), \tag{3.2}$$

where $I^{(0)}(\mathbf{x}_+')$ is the intensity distribution over this plane, and $\mu^{(0)}(\mathbf{x}_1'-\mathbf{x}_2')$ is the complex degree of spectral coherence between two points $\mathbf{x}_1'$ and $\mathbf{x}_2'$ on the plane at frequency ω. For a quasi-homogeneous source, we require that the intensity $I^{(0)}(\mathbf{x}_+')$ be a slow function relative to $\mu^{(0)}(\mathbf{x}_-')$, meaning that $\mu^{(0)}(\mathbf{x}_-')$ has a bounded domain of support and that $I^{(0)}(\mathbf{x}_+')$ must vary so slowly as to be essentially constant over any domain of equivalent area. The coordinates on the opposite sides of Equation 3.2 are related by

$$\begin{aligned}
x_+' &= \frac{(x_1'+x_2')}{2}, & y_+' &= \frac{(y_1'+y_2')}{2}, \\
x_-' &= (x_1'-x_2'), & y_-' &= (y_1'-y_2'), \\
x_1' &= x_+' + \frac{x_-'}{2}, & y_1' &= y_+' + \frac{y_-'}{2}, \\
x_2' &= x_+' - \frac{x_-'}{2}, & y_2' &= y_+' - \frac{y_-'}{2}.
\end{aligned} \tag{3.3}$$

Substituting from Equation 3.2 into Equation 3.1, we get

$$B_{\mathrm{MW}}^{(0)}(\mathbf{x},\mathbf{s}) = I^{(0)}(\mathbf{x})\,\tilde{\mu}^{(0)}(\mathbf{s})\cos\theta \tag{3.4}$$

for the radiance of the radiation from a quasi-homogeneous source, where the tilde again represents a two-dimensional Fourier transform of the form

$$\tilde{\mu}^{(0)}(\boldsymbol{\xi}) = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu^{(0)}(\mathbf{x})\exp(-ik\boldsymbol{\xi}\cdot\mathbf{x})\,d^2\mathbf{x}\,. \tag{3.5}$$

Equation 3.4 is equivalent to Equation 2.6 for the radiation from a quasi-homogeneous source.

Since both definitions we have used for the radiance function are equivalent for quasi-homogeneous sources, Equation 3.4 is also true for Walther's second definition of radiance, so that

$$B_{\mathrm{W}}^{(0)}(\mathbf{x},\mathbf{s}) = I^{(0)}(\mathbf{x})\,\tilde{\mu}^{(0)}(\mathbf{s})\cos\theta\,, \tag{3.6}$$

as shown specifically by Carter and Wolf[14] and as used extensively since then.[7,8] Thus Equation 2.12, which was derived in general for any type of radiation, can be used to study the propagation properties of the radiance function for quasi-homogeneous light as given by Equation 3.6. Before discussing this further, however, the significance of Equation 3.4 will be examined within the classical theory of radiometry.

The radiant intensity radiated from the $z = 0$ plane in the direction given by the unit vector $\mathbf{s}$ into the far field was found by Marchand and Wolf[11] to be given by

$$J_\omega(\mathbf{s}) = \cos\theta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B^{(0)}(\mathbf{x},\mathbf{s})\, d^2\mathbf{x}\,, \tag{3.7}$$

where for a quasi-homogeneous source we can now drop the subscript that denotes which of Walther's definitions we are using. Thus, substituting from Equation 3.4 into Equation 3.7, we find that

$$J_\omega(\mathbf{s}) = \lambda^2 \cos^2\theta\, \bar{I}^{(0)}(0)\, \tilde{\mu}^{(0)}(\mathbf{s})\,, \tag{3.8}$$

where

$$\bar{I}^{(0)}(0) = \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I^{(0)}(\mathbf{x})\, d^2\mathbf{x} \tag{3.9}$$

is a constant proportional to the total intensity integrated over the $z = 0$ plane.

Substituting from Equation 3.8 into Equation 3.4 for $\tilde{\mu}^{(0)}$, we have

$$B_{\mathrm{MW}}^{(0)}(\mathbf{x},\mathbf{s}) = \frac{I^{(0)}(\mathbf{x})}{\lambda^2 \bar{I}^{(0)}(0)}\, \frac{J_\omega(\mathbf{s})}{\cos\theta}\,. \tag{3.10}$$

From Equation 3.10, we see that this radiance function factors into a function of $\mathbf{x}$, which is proportional to the intensity distribution over the $z = 0$ plane; a function of $\mathbf{s}$, which is proportional to the radiant intensity radiated from the plane in the direction $\mathbf{s}$; and a $1/\cos\theta$ obliquity factor. Although Equations 3.10 and 2.6 each factor into a function of $\mathbf{s}$ and a function of $\mathbf{x}$, note that $\mathbf{s}$ and $\mathbf{x}$ are not Fourier conjugate variables in this theory for radiation from a quasi-homogeneous source as they are for a more general theory in which the quasi-homogeneous approximation is not used. From Equation 3.10, we see that both of Walther's definitions for radiance can, for the special case of quasi-homogeneous light, be interpreted as describing the flow of light energy from points in the $z = 0$ plane into specific directions toward the far field. This is in keeping with the usual classical idea of radiance.

To develop a wave theory for a non-imaging concentrator, we again consider the transformation of the radiance function from the $z' = 0$ plane to the $z = 0$ plane by an optical system, as illustrated in Figure 2.2; however, this time we will assume that the light in the input $z' = 0$ plane is quasi-homogeneous.

If the optical system shown in Figure 2.2 is a general, linear, stationary system, then we know that the cross-spectral density function over the $z = 0$ plane is given by[15]

$$\begin{aligned} W_\omega^{(z=0)}(\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{\lambda^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{x}'_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{x}'_2\, W_\omega^{(z'=0)}(\mathbf{x}'_1,\mathbf{x}'_2) \\ \times h(\mathbf{x}_1 - \mathbf{x}'_1)\, h^*(\mathbf{x}_2 - \mathbf{x}'_2) \end{aligned} \tag{3.11}$$

[15] W. H. Carter, "Generalization of Hopkins' Formula to a Class of Sources with Arbitrary Coherence," *J. Opt. Soc. Am. A: Opt. Image Sci.* **2**, 164–166 (1985).

as a function of the cross-spectral density function over the input $z'=0$ plane. Substituting from Equation 3.2 into Equation 3.11, we find that this system transforms the cross-spectral density function for the radiation from a quasi-homogeneous source into

$$W_{\omega}^{(z=0)}(\mathbf{x}_1,\mathbf{x}_2)=\frac{1}{\lambda^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{x}'_+ I^{(z'=0)}(\mathbf{x}'_+)$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{x}'_-\,\mu^{(z'=0)}(\mathbf{x}'_-)h\left(\mathbf{x}_1-\mathbf{x}'_+-\frac{\mathbf{x}'_-}{2}\right)$$
$$\times h^*\left(\mathbf{x}_2-\mathbf{x}'_++\frac{\mathbf{x}'}{2}\right). \tag{3.12}$$

From Equation 3.12, we see that the field over the output plane is not generally quasi-homogeneous.

There are some linear, stationary optical systems that can transform a quasi-homogeneous field over the $z'=0$ input plane into a quasi-homogeneous field over the $z=0$ output plane. Transforming coordinates in the kernel of the propagation integral in Equation 3.12 using Equation 3.3, we have

$$h\left(\mathbf{x}_1-\mathbf{x}'_x-\frac{\mathbf{x}'_-}{2}\right)h^*\left(\mathbf{x}_2-\mathbf{x}'_x+\frac{\mathbf{x}'_-}{2}\right)=h\left(\mathbf{x}_+-\mathbf{x}'_x+\frac{\mathbf{x}_--\mathbf{x}'_-}{2}\right)$$
$$\times h^*\left(\mathbf{x}_+-\mathbf{x}'_x-\frac{\mathbf{x}_--\mathbf{x}'_-}{2}\right). \tag{3.13}$$

Thus, we find from substitution of Equation 3.13 into Equation 3.12 that to produce a quasi-homogeneous field over the $z=0$ output plane the spread function must obey the equation

$$h\left(\mathbf{x}_++\frac{\mathbf{x}_-}{2}\right)h^*\left(\mathbf{x}_+-\frac{\mathbf{x}_-}{2}\right)=k_I(\mathbf{x}_+)k_\mu(\mathbf{x}_-). \tag{3.14}$$

Substituting Equation 3.14 into Equation 3.12, we find that the the two surface integrals in this equation can be separated so that the intensity transforms between the two planes as

$$I^{(z=0)}(\mathbf{x}_+)=\frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I^{(z'=0)}(\mathbf{x}'_+)k_I(\mathbf{x}_+-\mathbf{x}'_+)dx'_+\,dy'_+ \tag{3.15}$$

and the complex degree of spectral coherence transforms as

$$\mu^{(z=0)}(\mathbf{x}_-)=\frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu^{(z'=0)}(\mathbf{x}'_-)k_\mu(\mathbf{x}_--\mathbf{x}'_-)dx'_-\,dy'_-, \tag{3.16}$$

where the cross-spectral density function for the quasi-homogeneous field in the $z = 0$, output plane is given by

$$W_{\omega}^{(z=0)}(\mathbf{x}_1, \mathbf{x}_2) = I^{(z=0)}(\mathbf{x}_+)\,\mu^{(z=0)}(\mathbf{x}_-). \tag{3.17}$$

Hereafter, a linear, stationary optical system with a spread function that satisfies Equations 3.14, 3.15, and 3.16 will be referred to as a quasi-homogeneous preserving (QHP) system. An example of such a QHP system is the linear, stationary optical system with a Gaussian spread function given by

$$h(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left[-|\mathbf{x}|^2/\left(2\sigma^2\right)\right]. \tag{3.18}$$

Substituting from Equation 3.18 into Equation 3.14, we find that

$$k_I(\mathbf{x}) = \frac{1}{\pi\sigma^2} \exp\left(-|\mathbf{x}|^2/\sigma^2\right), \tag{3.19}$$

and

$$k_\mu(\mathbf{x}) = \frac{1}{4\pi\sigma^2} \exp\left[-|\mathbf{x}|^2/\left(4\sigma^2\right)\right]. \tag{3.20}$$

Thus, any linear, stationary optical system with a Gaussian spread function is a QHP system.

We can derive an equation equivalent to Equation 2.12 for the transfer of the radiance function of the radiation from a quasi-homogeneous source through a QHP optical system. Substituting from Equations 3.15 and 3.16 into Equation 3.4 for the radiance function over the z=0 plane, and again using Equation 3.4 to represent the radiance function over the $z' = 0$ plane, we get

$$B_{\mathrm{MW}}^{(z=0)}(\mathbf{x}, \mathbf{s}) = \left[B_{\mathrm{MW}}^{(z'=0)}(\mathbf{x}, \mathbf{s}) * k_I(\mathbf{x})\right]\tilde{k}_\mu(\mathbf{s}), \tag{3.21}$$

where the tilde indicates a Fourier transform:

$$\tilde{k}_\mu(\mathbf{s}) = \frac{1}{\lambda^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_\mu(\mathbf{x}) \exp\left[-ik(\mathbf{s}\cdot\mathbf{x})\right] d^2\mathbf{x}. \tag{3.22}$$

Equation 3.21 is identical to Equation 2.12, which was obtained for a light field of arbitrary coherence, except that $\tilde{k}_\mu(\mathbf{s})$ is not the Fourier transform of $k_I(\mathbf{x})$, and the two Fourier kernels are missing from Equation 3.21.

The minimum spot size a QHP concentrator can produce with radiation from any quasi-homogeneous source can also be calculated. Because the field in both the input and output planes is quasi-homogeneous, we have in each of these planes an intensity function that is slow relative to the complex degree of spectral coherence, so that

$$\begin{aligned} &\Delta x_+ >> \Delta x_- , \quad \Delta y_+ >> \Delta y_- , \\ &\Delta x'_+ >> \Delta x'_- , \quad \Delta y'_+ >> \Delta y'_- , \end{aligned} \tag{3.23}$$

where Δx_+ and Δy_+ are the standard deviations of $I^{(z=0)}(\mathbf{x}_+)$ in respect to x_+ and y_+; Δx_- and Δy_- are the standard deviations of $\mu^{(z=0)}(\mathbf{x}_-)$ in respect to x_- and y_-; $\Delta x'_+$ and $\Delta y'$ are the standard deviations of $I^{(z'=0)}(\mathbf{x}'_+)$ in respect to x'_+ and y'_+; and $\Delta x'_-$ and $\Delta y'_-$ are the standard deviations of $\mu^{(z'=0)}(\mathbf{x}'_-)$ in respect to x'_- and y'_-. From Equations 3.4 and 3.5, we see that s is a Fourier conjugate variable to $\mathbf{x}_-$, and s′ is a Fourier conjugate variable to $\mathbf{x}'_-$. Thus, from the uncertainty principle for Fourier conjugate functions, we have

$$\begin{aligned} &\Delta x_- \Delta s_x \geq \frac{\lambda}{2\pi} , \quad \Delta y_- \Delta s_y \geq \frac{\lambda}{2\pi} , \\ &\Delta x'_- \Delta s'_x \geq \frac{\lambda}{2\pi} , \quad \Delta y'_- \Delta s'_y \geq \frac{\lambda}{2\pi} , \end{aligned} \tag{3.24}$$

where Δs_x and Δs_y are the standard deviations of the function $\tilde{\mu}^{(z=0)}(\mathbf{s})$ relative to s_x and s_y, and $\Delta s'_x$ and $\Delta s'_y$ are the standard deviations of the function $\tilde{\mu}^{(z'=0)}(\mathbf{s}')$ relative to s'_x and s'_y. From Equations 3.20 and 3.21, we have

$$\begin{aligned} &\Delta x_+ >> \Delta x_- \geq \frac{\lambda}{2\pi \Delta s_x} , \\ &\Delta y_+ >> \Delta y_- \geq \frac{\lambda}{2\pi \Delta s_y} , \\ &\Delta x'_+ >> \Delta x'_- \geq \frac{\lambda}{2\pi \Delta s'_x} , \\ &\Delta y'_+ >> \Delta y'_- \geq \frac{\lambda}{2\pi \Delta s'_y} . \end{aligned} \tag{3.25}$$

Thus, the smallest spot that can ever be achieved with radiation from a quasi-homogeneous source is limited by the equation

$$S_{QH} \stackrel{\Delta}{=} \mathrm{Min}(\Delta x_+ \, \Delta y_+) >> \left(\frac{\lambda}{2\pi}\right)^2 \tag{3.26}$$

for the maximum values of $\Delta s_x \approx \Delta s_y \approx 1$. Comparing Equation 3.26 with Equation 2.16, we see that radiation from a quasi-homogeneous source (such as a thermal source) can never be concentrated as much as light from some non–quasi-homogeneous sources (such as a laser source).

Although Equation 3.26 gives a basic limitation on the smallest possible spot of light that can be formed with radiation from a kernel source, the second law of thermodynamics gives another, independent, limitation on what can be done to concentrate a given radiation field from a quasi-homogeneous source. The concentration, as defined in Equation 2.19, can be applied to radiation from a quasi-homogeneous source to obtain the optimum concentration that can be obtained by a QHP concentrator. To obtain this limitation, we again use the second law of thermodynamics, as in section 2:

$$\frac{V^{(\text{in})}}{V^{(\text{out})}} \triangleq \frac{\Delta x'_+ \Delta y'_+ \Delta s'_x \Delta s'_y}{\Delta x_+ \Delta y_+ \Delta s_x \Delta s_y} \leq 1, \tag{3.27}$$

so that upon substitution from Equation 3.27 into Equation 2.19, we get

$$C = \frac{\Delta x'_+ \Delta y'_+}{\Delta x_+ \Delta y_+} \leq \frac{\Delta s_x \Delta s_y}{\Delta s'_x \Delta s'_y}, \tag{3.28}$$

which is equivalent to Equation 2.21. If we assume, as before, that the optical system and the illumination are symmetrical about the z axis in both spaces, so that $\Delta s'_x = \Delta s'_y = \sin\theta'$ and $\Delta s_x = \Delta s_y = \sin\theta$, then we find that

$$C \leq \left(\frac{\sin\theta}{\sin\theta'}\right)^2, \tag{3.29}$$

so that the absolute limit on the concentration is given again by

$$C \leq \frac{1}{\sin^2\theta'}, \tag{3.30}$$

for the case where $\sin\theta$ approaches unity.

As with the more general case, we have two independent limitations on the concentration. Equation 3.26 gives a fundamental (diffraction) limitation on the concentration of radiation from all quasi-homogeneous sources, which is that it can never be as well concentrated as some other radiation fields. Equations 3.28 and 3.30 give another limitation, which is that radiation from a given quasi-homogeneous source can not be concentrated more than the ratio of numerical apertures, before and after concentration, without violating the second law of thermodynamics.

Since both of Walther's definitions for radiance are equivalent for quasi-homogeneous light, there is another approach for studying the propagation of the radiance function for quasi-homogeneous light through the linear, stationary optical system shown in Figure 2.2. Substituting from Equation 3.6 into Equation 2.12, we find that the radiance function for the field in the output plane of any linear, stationary optical system is given by

$$B^{(z=0)}(\mathbf{x},\mathbf{s}) = \frac{1}{\lambda^2}\tilde{\mu}^{(0)}(\mathbf{s})\cos\theta\exp(ik\mathbf{s}\cdot\mathbf{x})\tilde{h}(s_x,s_y) \times\left\{\left[I^{(0)}(\mathbf{x})\exp(-ik\mathbf{s}\cdot\mathbf{x})\right] * h^*(\mathbf{x})\right\}, \tag{3.31}$$

provided that the light over the input plane is quasi-homogeneous. Consider the factor in braces given by

$$F(\mathbf{x},\mathbf{s}) \stackrel{\Delta}{=} \frac{1}{\lambda^2}\left\{\left[I^{(0)}(\mathbf{x})\exp(-ik\mathbf{s}\cdot\mathbf{x})\right] * h^*(\mathbf{x})\right\}$$

$$= \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I^{(0)}(\mathbf{x}')h^*(\mathbf{x}-\mathbf{x}')\exp(-ik\mathbf{s}\cdot\mathbf{x}')\, d^2\mathbf{x}' . \tag{3.32}$$

If we assume that the spread function $h(\mathbf{x})$ has a domain of support about the origin that is very small relative to any domain over which $I^{(0)}(\mathbf{x})$ varies appreciably, so that $I^{(0)}(\mathbf{x})$ is a slow function relative to $h(\mathbf{x})$, then Equation 3.32 can be approximated by

$$F(\mathbf{x},\mathbf{s}) = I^{(0)}(\mathbf{x})\tilde{h}^*\left(s_x, s_y\right)\exp(-ik\mathbf{s}\cdot\mathbf{x}) , \tag{3.33}$$

where the tilde denotes the Fourier transform defined by Equation 2.11. Substituting from Equation 3.33 back into Equation 3.30, we have

$$B^{(z=0)}(\mathbf{x},\mathbf{s}) = \tilde{\mu}^{(0)}(\mathbf{s})I^{(0)}(\mathbf{x})\cos\theta|\tilde{h}(\mathbf{s})|^2 , \tag{3.34}$$

which describes the manner in which the radiance function propagates through any linear, stationary optical system for which the spread function satisfies the approximation that $I^{(0)}(\mathbf{x})$ is a slow function relative to $h(\mathbf{x})$, leading to Equation 3.33.

Foley and Wolf[7,8] have shown that if the optical system is simply propagation through free space from one plane to another, Equation 3.34 is closely approximated, in a short-wavelength limit, by

$$B_{\mathrm{W}}^{(z=0)}(\mathbf{x},\mathbf{s}) = \tilde{\mu}^{(0)}(\mathbf{s})I^{(0)}(\mathbf{x})\cos\theta , \tag{3.35}$$

so that the radiance function is the same in both the input and output planes and is therefore invariant upon propagation. From Equation 3.34, it is clear that for propagation of the radiance function through an optical system that has a unimodular spread function $\tilde{h}(\mathbf{s})$, so that

$$|\tilde{h}(\mathbf{s})|^2 = 1 \tag{3.36}$$

over the domain where $|\mathbf{s}| \leq 1$, Equation 3.35 still holds. Thus, there is a whole class of optical systems for which the radiance function is invariant.

The free space propagation of a field that contains no evanescent plane waves has a transfer function given by

$$\tilde{h}(\mathbf{s}) = \exp\left[ik\sqrt{1-s_x^2-s_y^2}\,(z-z')\right], \tag{3.37}$$

which is unimodular. But the corresponding spread function is the Rayleigh kernel, which does not have bounded support. Thus, the approximation that was used to obtain Equation 3.33 does not hold for this case. Nevertheless, Foley and Wolf have shown that Equation 3.35 does hold for free space propagation, provided the wavelength is sufficiently small. The radiance function for light is therefore invariant for both free space propagation and propagation through a large class of optical systems.

4. CONCLUSIONS

This report has considered the concentration of a light field in any state of coherence by any optical system. The special case of the concentration of radiation from quasi-homogeneous sources, which is applicable to radiation from thermal sources and most other naturally occurring sources of low coherence, has also been considered.

A simple equation has been derived for propagating the radiance function through any linear, stationary concentrator. This relation should be useful for modeling such concentrators when a wave theory is required. A somewhat simpler equation for describing the propagation of the radiation from a quasi-homogeneous source through a QHP concentrator has also been found, which reproduces a quasi-homogeneous source field in the output plane. Finally, an approximate expression has been obtained for the propagation of the radiance function for quasi-homogeneous light through any linear, stationary optical system. This relation shows that for a large class of systems, including free space propagation, the radiance function is an invariant.

It was possible to derive very general limitations on the smallest possible spot of light that can be achieved by any light concentrator. For monochromatic components of a light field, the smallest possible spot is given by Equation 2.16. This is simply the well-known diffraction limit for focusing any monochromatic wave field. For radiation from quasi-homogeneous sources, it was found in Equation 3.26 that the smallest possible spot must be much bigger than the diffraction limit.

Finally, it was shown that the well-known limitation on the concentration that can be achieved with a given light source can be derived for any concentrator and any type of light field from a wave theory, and is a requirement of the second law of thermodynamics.

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