Research Note
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Ambiguity in Calculating and Interpreting Maximum Drawdown

Introduction

We all know what maximum drawdown is. According to the CFA Institute, maximum drawdown is defined as "the maximum loss from a market peak to a market nadir". This may sound reasonable and clear, but as we will see, the ambiguity already starts here.

We define ***drawdown*** as the difference between the portfolio value and its running maximum; ***maximum drawdown*** is therefore the largest drawdown. The running maximum is also known as the ***high water mark***. Figure 1 illustrates the high water mark and maximum drawdown for a specific time series of portfolio values.

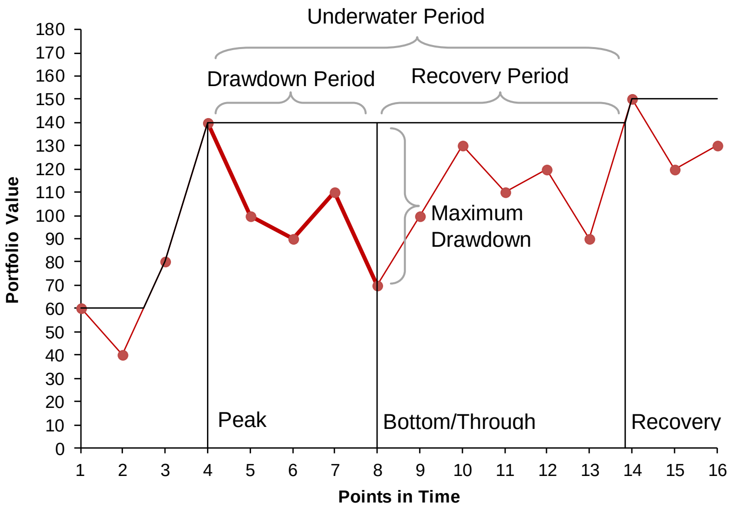

Figure 1 - Maximum Drawdown

The time period from the peak at point 4 and the bottom in point 8 is called the ***drawdown period***; the time period from the bottom point 8 back up to the levels of the previous peak in point 14 is the ***recovery period***. The time period between point 4 and 14 can be called the ***underwater period***. Some people do not differentiate between drawdown period and underwater period, but use drawdown period to refer to the period between points 4 and 14.

The terminology we use is not written in stone. The point we are trying to make is that all the elements in Figure 1 and discussed above should be defined in a clear and unambiguous manner when analyzing maximum drawdowns.

Maximum drawdown (MDD) is a relatively novel risk measure. Its origins in the scientific finance literature can be traced back to a paper by Zhongquan Zhou and Sanford J. Grossmann in 1993 concerned with the following issue: "We analyze the optimal risk investment policy for an investor who, at each point in time, wants to lose no more than a fixed percentage of the maximum value his wealth has achieved up to that time. In particular, if M(t) is the maximum level of wealth W attained on or before time t, then the constraint imposed on his portfolio choice is that W(t) >= a*M(t), where a is an exogenous number between 0 and 1." The difference between W(t) and M(t) is what we call drawdown. Zhaongquan/Grossmann concluded that the optimal investment strategy in the presence of drawdown constraints is similar to the constant proportion portfolio insurance (CPPI) proposed by Black and Perold in 1987, with the difference that the floor is not a exogenous constant, but a path-dependent, stochastic variable.

MDD is usually expressed as a percentage figure calculated like a simple investment return: (bottom value – peak value) / peak value. In our example: MDD = (70-140)/140 = -50%. While it is common practice to «annualize» the figures when comparing investment returns, MDD figures are left as calculated. This means that contrary to common practice in performance analysis, one might be comparing returns measured over different time periods. Maximum drawdown therefore is a loss measure ignoring the time period over which the losses were accumulated.

Maximum Loss Statistics

MDD is very often confused with other "maximal" loss measures.

One example is "maximum loss", the smallest period return. The maximum period loss in our example is -36% and occurred between points 7 and 8 as shown in Figure 2 below.

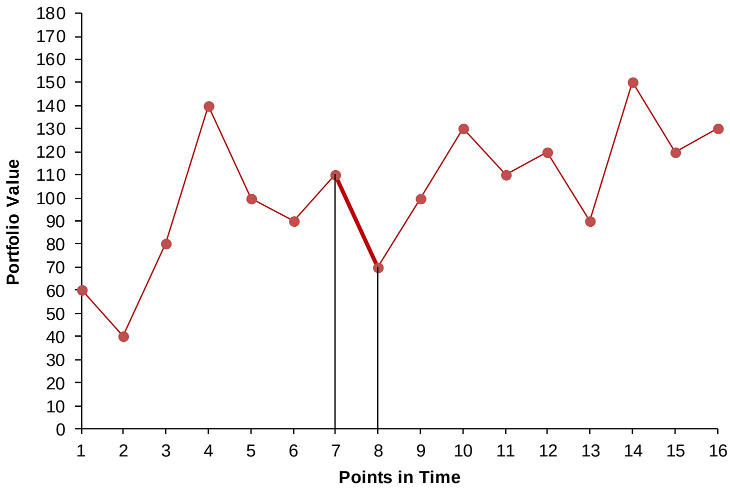

Figure 2 - Maximum Loss

MDD is cumulative loss, not a one-period loss. Further, MDD is a loss relative to the high water mark, not the period beginning value.

Many are also tempted to calculate maximum drawdown as the percentage difference between the highest and the lowest portfolio value in a series, which would be 73.6% in our example. This is equivalent to calculating the difference between the global maximum and global minimum of a series. The result will generally be different from maximum drawdown, which is a difference between a local maximum and a local minimum in a series. As this calculation ignores any time aspects, results can be absurd. This is the case in our example portfolio, in which the highest portfolio value (point 14) occurs after the lowest one (point 2).

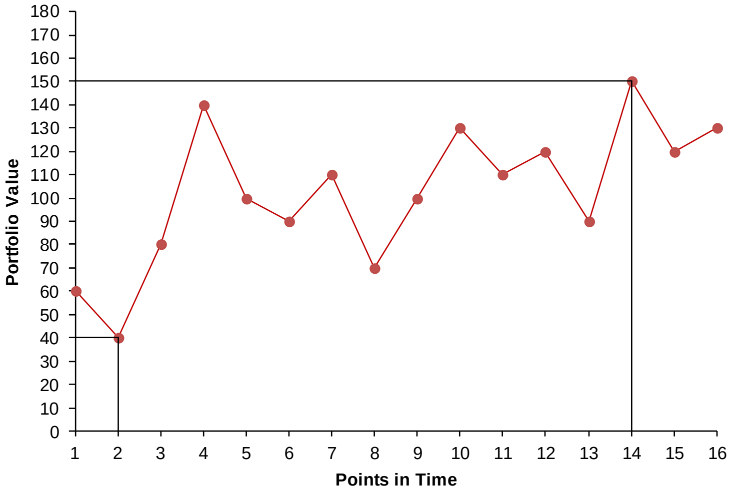

Figure 3 - Global Maximum and Global Minimum

MDD is the largest difference between portfolio value and high water mark, not the difference between the highest and lowest portfolio value.

Losing Runs

Losing and winning runs are expressions commonly used in a gambling context. A losing run is defined as cumulative consecutive losses. In our example, the maximum drawdown is a not a losing run because a partial and temporary recovery takes place between points 6 and 8…

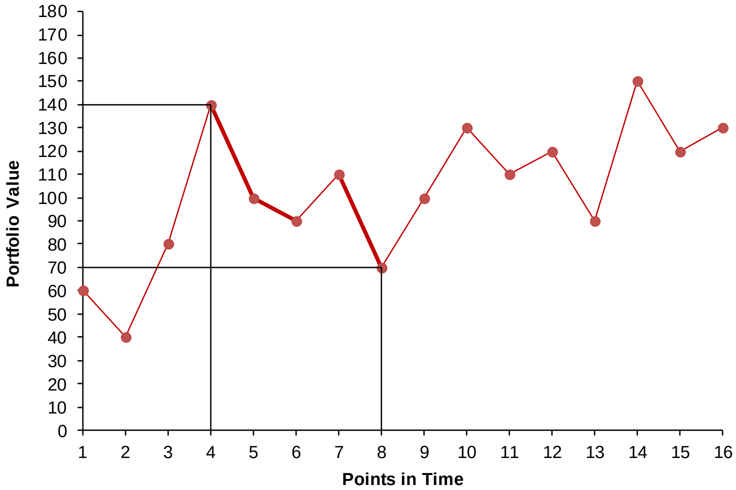

Figure 4 - Losing Runs

While losing runs contain interesting information about the underlying stochastic process generating portfolio values, they are difficult to reconcile with realistic risk preferences of investors. For example, an investor in point 8 will probably be more concerned with the drop from 140 to 70 resulting in a -50% drawdown rather than his largest losing run, which would be the -36.4% move from point 7 to point 8.

MDD is the cumulative loss across partial and temporary recoveries, not just consecutive losses.

Shortfall Risk

Shortfall risk is about falling short of a given threshold. Shortfall risk can be expressed as the probability of not achieving the threshold or as the loss relative to the threshold incurred. Shortfall is typically used as an end-of-period risk measure: “Given a certain investment horizon between two points in time, what are the shortfall risks at the end of the investment horizon?” The threshold is determined by the investment goals and risk preferences of the investor.

In a backward looking context, shortfall probability is a Boolean variable: looking at the portfolio at the end of the investment period, we have either fallen short of the threshold or not. The shortfall is calculated as the difference between the end-of-period portfolio value and the threshold. Assuming a threshold of 135, our example portfolio would have fallen short (eop portfolio value is 130), incurring a shortfall loss of (130-135)/135 = -3.7%.

MDD is conceptually very different in the following ways:

- MDD considers all portfolio values during the investment period (“interim risk”), and does not just compare ending and beginning portfolio values.
- MDD is measured relative to a path-dependent threshold, i.e. the high water mark. High water marks are not known in advance with certainty; they only reflect asset characteristics and not investor risk preferences.

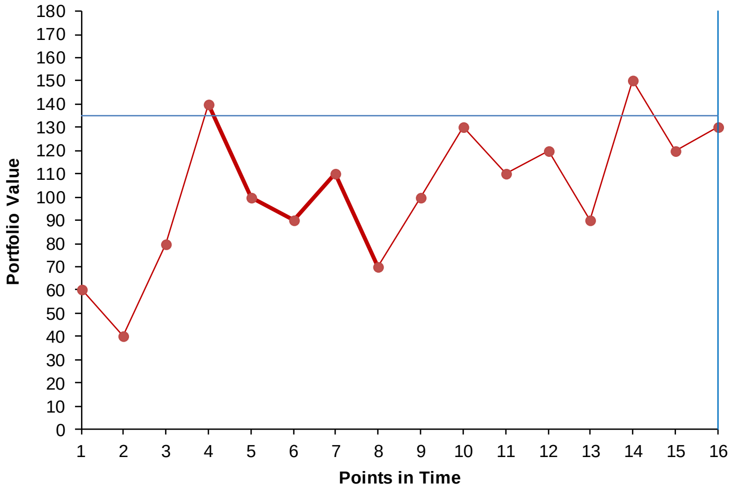

Figure 5 - Interm Risk Versus End-Of-Period Risk

MDD is a path-dependent risk measure considering interim losses.

Downside Risk

In a forward looking context shortfall risks are calculated from an estimated or assumed return distribution. Returns from such distributions are assumed to cover the same time period, from the beginning to end of the investment period. There are a multitude of measures to express the risk characteristics of a distribution. The traditional method is to look at the second moment of the distribution, its standard deviation also known as the “volatility of returns”. Volatility measures the dispersion of returns around its mean. As dispersion includes positive and negative deviations from the mean, positive deviations contribute to risk as much as negative deviations. In real-world applications, dispersion risk is relevant to a very small number of investors. Most investors perceive risk as “downside”. A very popular downside risk measure is “Value-At-Risk” (VaR), defined as the loss over a certain time period that is not exceeded with a certain probability.

A discussion of the ambiguities surrounding VaR would probably be worth a separate research note. Nevertheless, VaR is a quantile loss measure, summarizing the risk characteristics of a return distribution in one value taken from that distribution. Similar to the shortfall risk measures, VaR only considers end-of-period portfolio values and not their path during the investment period.

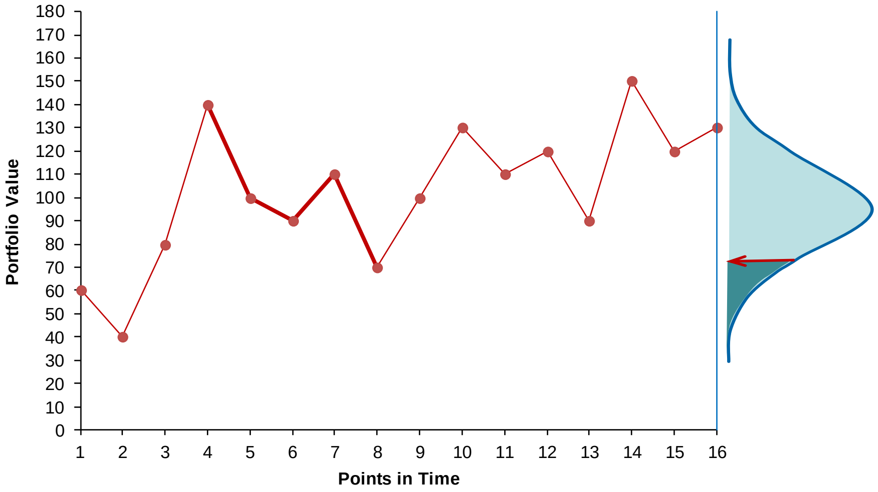

Figure 6 - Value-At-Risk

Only under very simplistic assumptions will we find a stable relationship between maximum drawdown and end-of-period VaR.

Given real-world features of asset return time series such as serial correlation, volatility clustering, fat tails and skewness, two portfolios with a similar VaR can exhibit very different drawdown characteristics.

Nth Largest Drawdowns

MDD is one drawdown amongst many others. We are often not only interested in the largest drawdown, but the second, third, …, nth largest drawdown or then summary characteristics of all drawdowns like for example average (“expected”) drawdowns. This is the case when calculating risk-adjusted performance measures like the Calmar or Burke Ratio.

We can calculate all drawdown figures as the relative difference of the current portfolio value from the high water mark and graph them in the so called "underwater chart" in figure 7 below.

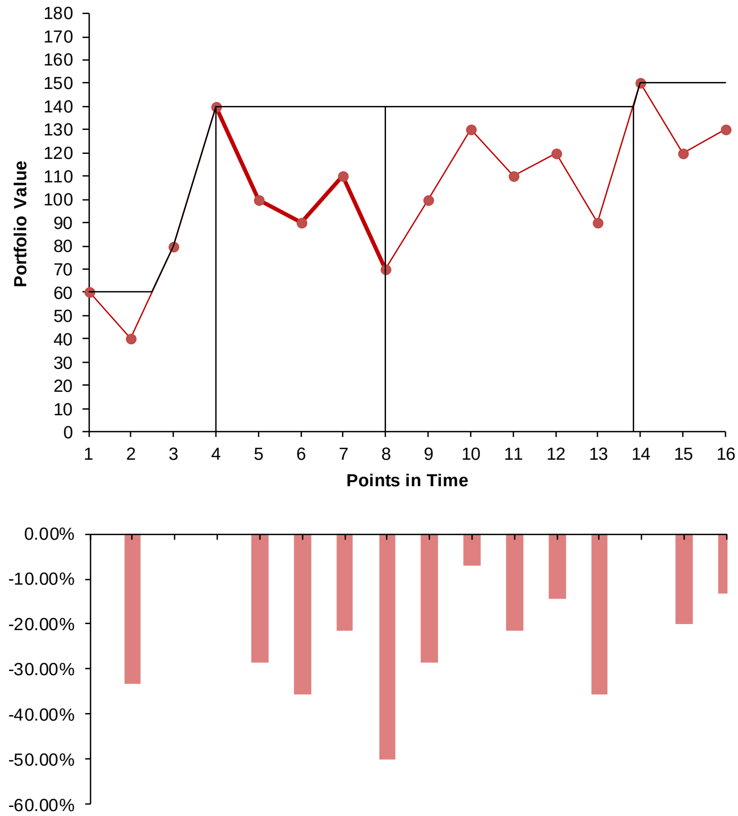

Figure 7 – Underwater Chart

There are several possibilities to determine the second largest drawdown.

But first of all, note that there exist two drawdowns of equal magnitude in this example: The drawdowns in points 6 and 13 are both -35.7%. The treatment of ties will have an impact on how we determine the third largest drawdown (if we decide it exists at all).

Both second largest drawdowns have the feature that they occur during the underwater period of the maximum drawdown. Point 6 occurs during the drawdown phase, while point 13 occurs during the recovery phase.

One can now argue that drawdowns should be determined based on "non-overlapping" time periods in the sense that a new drawdown can only occur if the other is "finished". But when exactly is a drawdown "finished"? Two types of "non-overlapping" time concepts can be distinguished:

1. We can set up the rule that the second largest drawdown is the second largest drawdown occurring outside the *drawdown* period of the maximum drawdown. If we do this, point number 13 is our choice.
2. Alternatively, we can say that the second largest drawdown is defined as the second largest drawdown occurring outside the *underwater* period of the maximum drawdown. In out example, this would be point number 2.

Choosing between the two concepts is difficult as time aspect are irrelevant by definition. With the first concept, the focus would remain on the “downside characteristics” as we are only considering the journey from peak to bottom. The second concept takes into account the recovery period, which would be more appropriate in a context of “underwater risk”.

A maximum drawdown methodology does not determine the identification of smaller drawdowns. Further methodological choices are necessary to calculate the nth drawdown in a systematic manner.

Conclusion

Maximum drawdown is often marketed as a risk measure “better aligned with risk preferences of real-world investors” than other risk measures like for example volatility. But “more realistic” does not necessarily mean “more intuitive”. As we have seen, maximum drawdown is an indicator for very specific risk features of portfolio or asset time series and complements other risk measures (like shortfall and downside risk) rather than replacing them. Subtle differences in calculation methodologies are not merely “technical details”, but differing ways of modeling risk. A single best method does not exist; we recommend working with explicit definitions that are aligned with preferences of the investors involved. We recommend investors that base investment decisions on maximum drawdown characteristics to ask for details regarding the calculation methodologies used.

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